

Interferometry II: Calibration and Imaging (CRASIES Day 2)

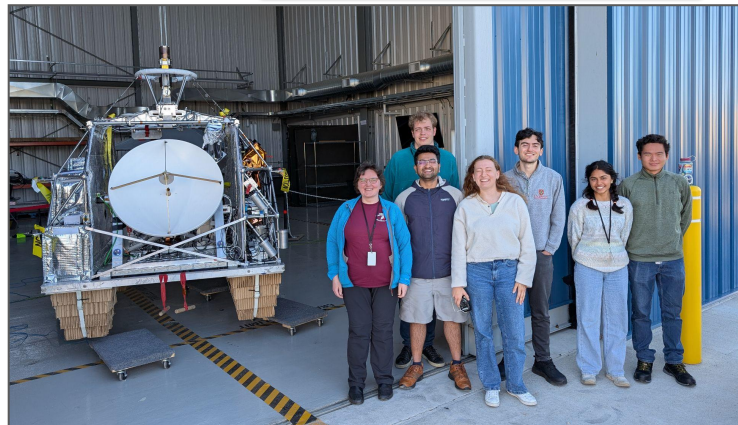
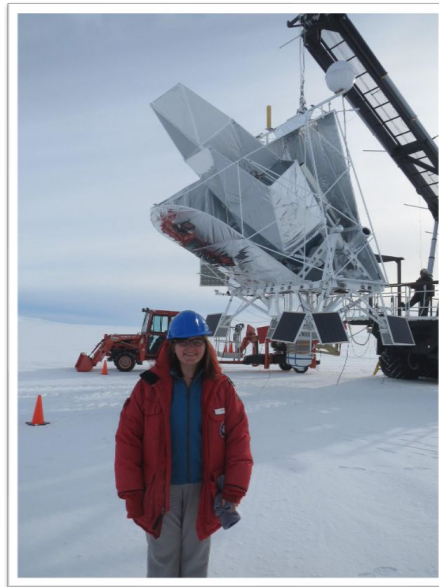


Laura Fissel
Associate Professor, Queen's University

Introduction

- 2000-2005: BSc in Physics and Astronomy and Co-op student at the University of Victoria
- 2006-2013: PhD University of Toronto
 - Helped build a balloon borne telescope BLASTPol that launched from Antarctica in 2010 and 2012
- 2013-2019: Postdoctoral fellowships at Northwestern University and the National Radio Astronomy Observatory
- September 2019: Started as an assistant professor at Queen's University.

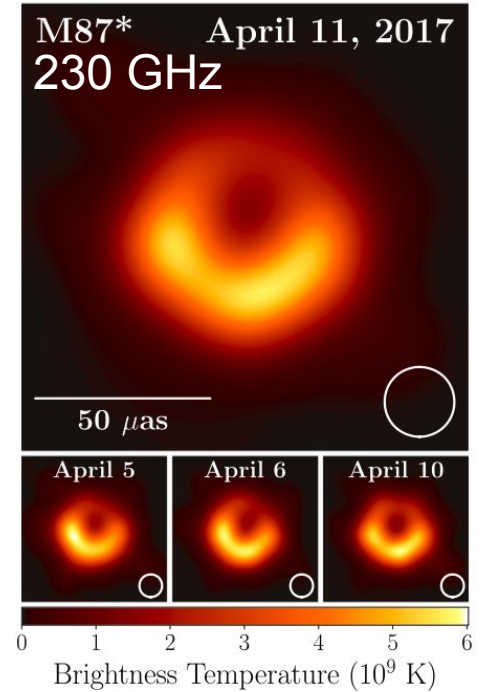
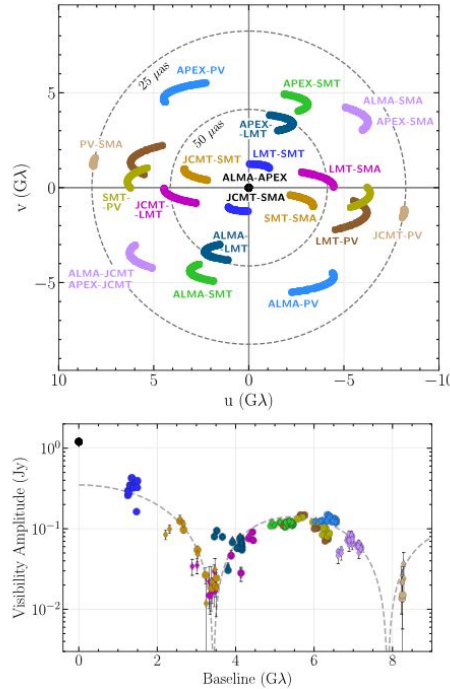
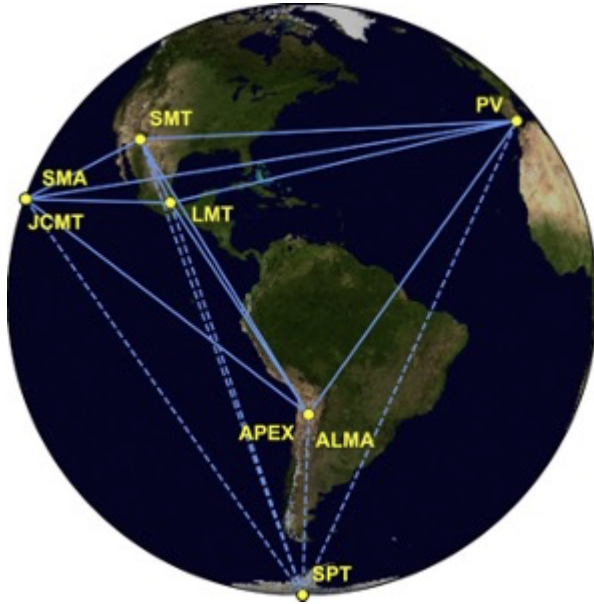
with BLASTPol
In 2012



Balloon-VLBI Experiment (BVEX) team in Timmins, ON 2025

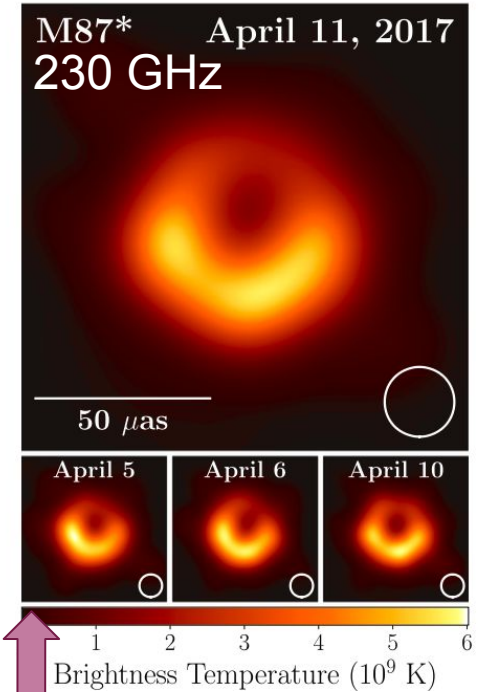
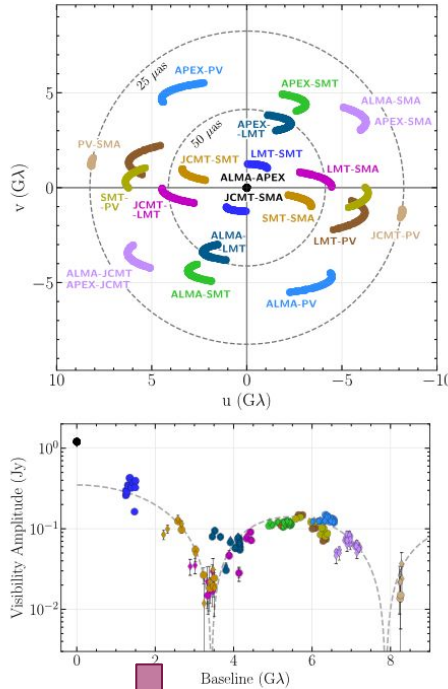
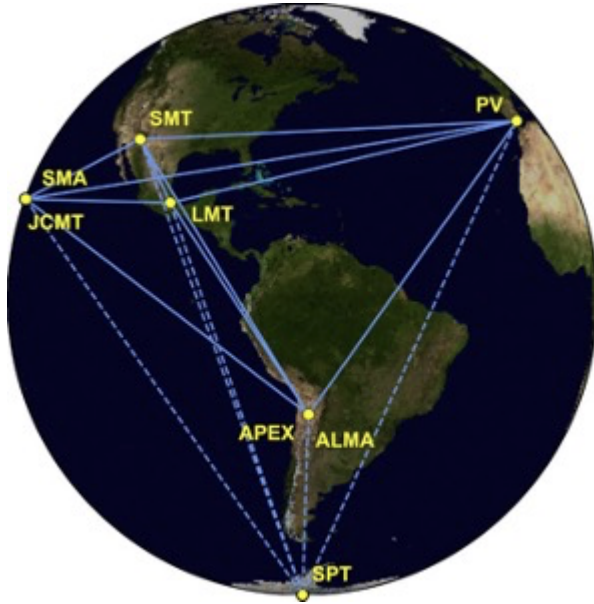
Where we left off in the last lecture... visibilities!

Example: EHT observations of the supermassive black hole at the centre of M87



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Plan for this lecture

Calibration: how do we convert our raw visibilities into calibrated flux measurements?

Model Fitting: Analyzing visibility data without imaging

Imaging: How do we make images from our data and understand their limitations?

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Model Fitting: Analyzing visibility data without imaging

Imaging: How do we make images from our data and understand their limitations?

References:

Interferometry and Synthesis in Radio Astronomy (3rd Edition) Thompson, Moran and Swenson: [Open Access Book](#)

NRAO's Synthesis Imaging Workshop Slides 2018:

- <https://science.nrao.edu/science/meetings/2018/16th-synthesis-imaging-workshop/16th-synthesis-imaging-workshop-lectures>
- Particularly [Calibration](#) by George Moellenbrock and [Imaging](#) by David Wilner

Calibration of Visibilities

Goals:

- Convert visibilities from raw instrumental units to flux units.
- Flag and remove bad data so that it is not included in your science analysis!
- Correct for instrumental and atmospheric effects.
 -

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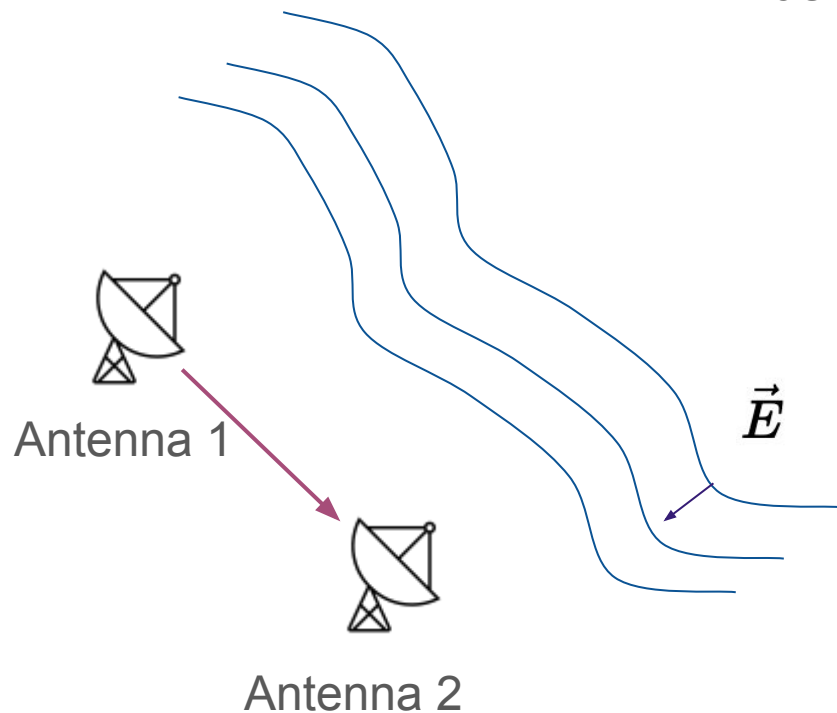
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Categories:

- **Slow changing**
 - E.g. antenna position, pointing or gain corrections with elevation, shadowing.
 - Can be characterized then applied to all observations.
- **Fast changing**
 - Variable phases delay from the atmosphere, fluctuations in system temperature, phase drifts in the LO.
 - Must be monitored during science observations by observing calibrators.

Complex Antenna Gains for Two Antennas:

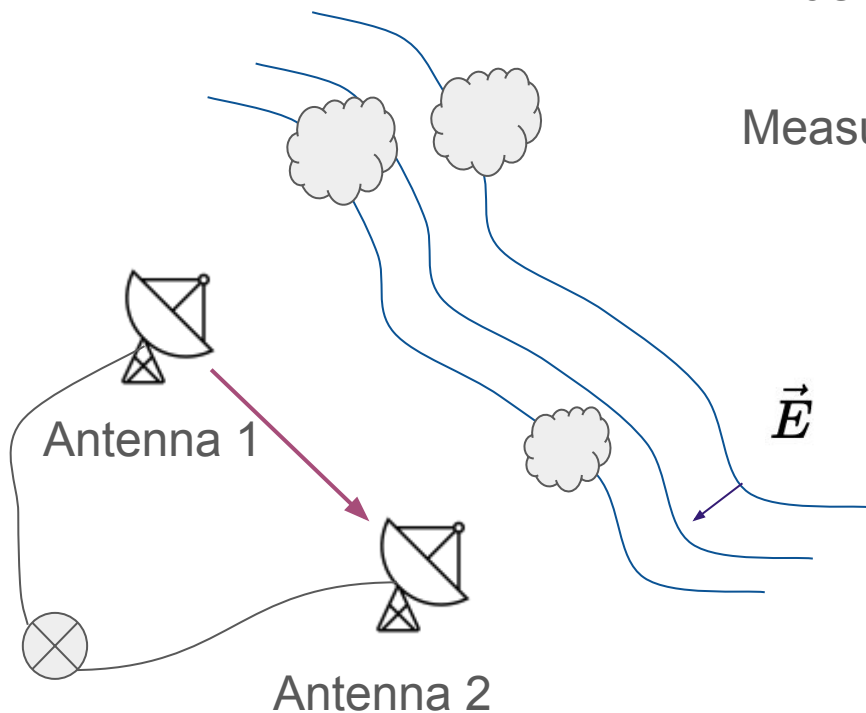
“True” visibility $V_{12} = \langle E_1 E_2^* \rangle = A_{12} e^{2\pi i \phi_{12}}$



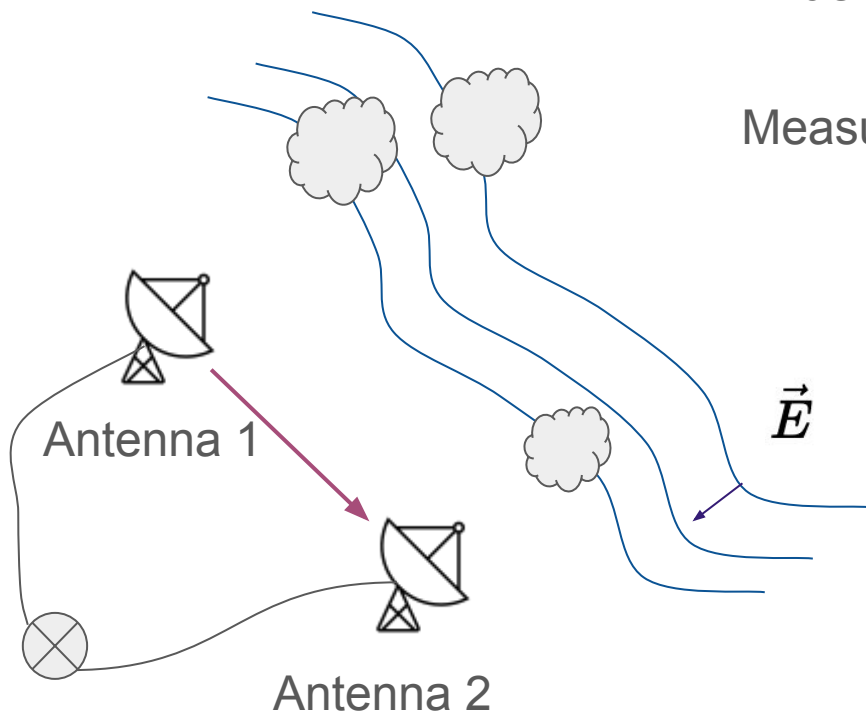
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Measured visibility $\hat{V}_{12} = \langle (G_1 E_1)(G_2 E_2)^* \rangle$



Complex Antenna Gains for Two Antennas:



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$$\begin{aligned} \text{Measured visibility } \hat{V}_{12} &= \langle (G_1 E_1)(G_2 E_2)^* \rangle \\ &= G_1 \langle E_1 E_2^* \rangle G_2^* \\ &= G_1 V_{12} G_2^* = G_{12} V_{12} \\ &= g_1 g_2 e^{2\pi i (\theta_1 - \theta_2)} V_{12} \end{aligned}$$

Calibrating visibilities

$$\hat{V}_{mn}(u, v) = G_{mn}(t) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I(l, m) e^{-2\pi i(ul + vm)} dl dm = G_{mn}(t) V_{mn}(t)$$

Uncalibrated visibilities
(what we measure)

Complex gain for
antenna pair m,n
(which varies with time)

Source intensity
distribution (what we
want to measure)

“Calibrated” visibility
for antenna m,n

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Basic calibration strategy: observe an object with known $I(l, m)$ and solve for $G_{mn}(t)$.

We want our calibrator to be:

- Bright (high signal to noise in a short integration time)
- Nearby your source (same path length through the atmosphere)
- Well characterized (known flux) and monitored (if time variable)

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Question to discuss at your table: which would be a better calibrator for tracking antenna gains: a bright point-source or a bright calibrator with extended structure? Why?

Calibration with a point source

$$\hat{V}_{mn}(u, v)_c = G_{mn}(t) S_c \Rightarrow V_{mn}(u, v) = \hat{V}_{mn}(u, v) \left[\frac{S_c}{\hat{V}_{mn}(u, v)_c} \right]$$

Measured visibility of
point source
calibrator c

Complex gain for
antenna pair m,n

Flux density of
calibrator in Jy

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Measured visibility of
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Flux density of
calibrator in Jy

Usually we calibrate by phase and amplitude separately:

Gain Amplitude: $|G_{mn}| = g_m g_n = \frac{S_c}{|\hat{V}_{mn c}|}$

Solve for the antenna gain amplitudes $\{g_m\}$,
and phases $\{\theta_m\}$.

Gain Phase: $\hat{\phi}_{mn c} = \theta_m - \theta_n + \cancel{\phi_{mn c}}$

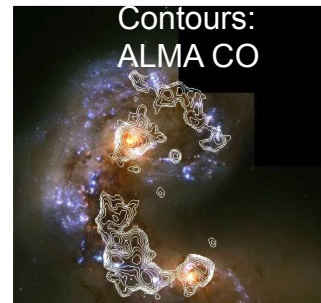
= 0 if the calibrator is a point
source at the phase centre

To recap we want our calibrator to be:

- Bright (high signal to noise in a short integration time)
- Nearby your source (same path length through the atmosphere)
- Well characterized (known flux) and monitored (if time variable)
- A point source (if possible)
- Well characterized polarization (if you are attempting a polarization observation)

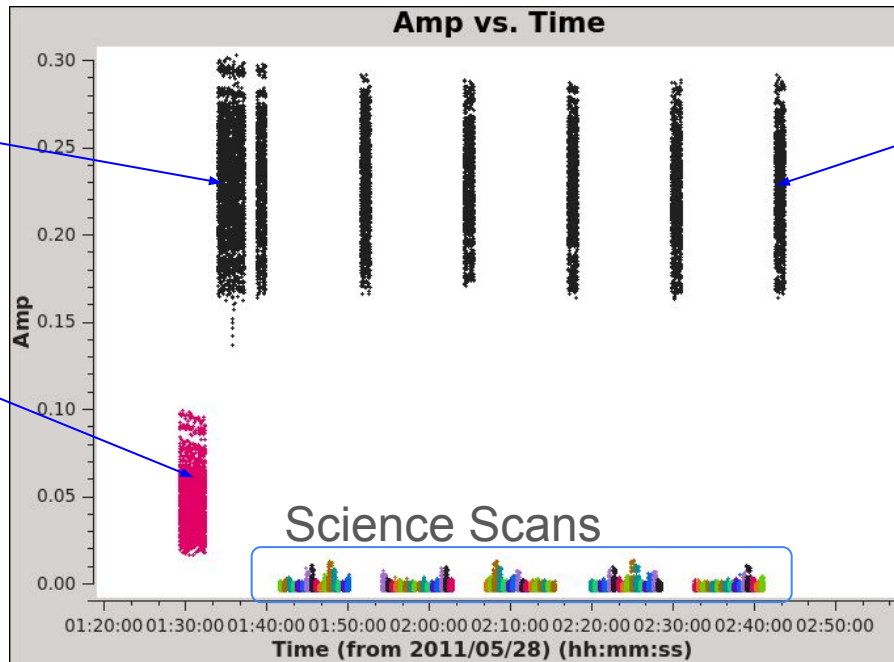
In practice it is very hard to find one calibrator that satisfies all these requirements so we observe more than one calibrator.

Example: ALMA Band 7 Observations of the Antennae



Band Pass Calibrator:
3c279

➤ Calibrate $G_{mn}(\nu)$



Gain Calibrator: 3c279

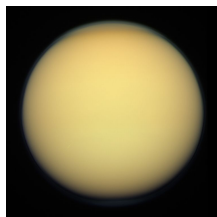
➤ Track gain changes in time

18.6° from Antennae

Observed every 15 minutes to track changes in complex gain

Flux Calibrator: Titan

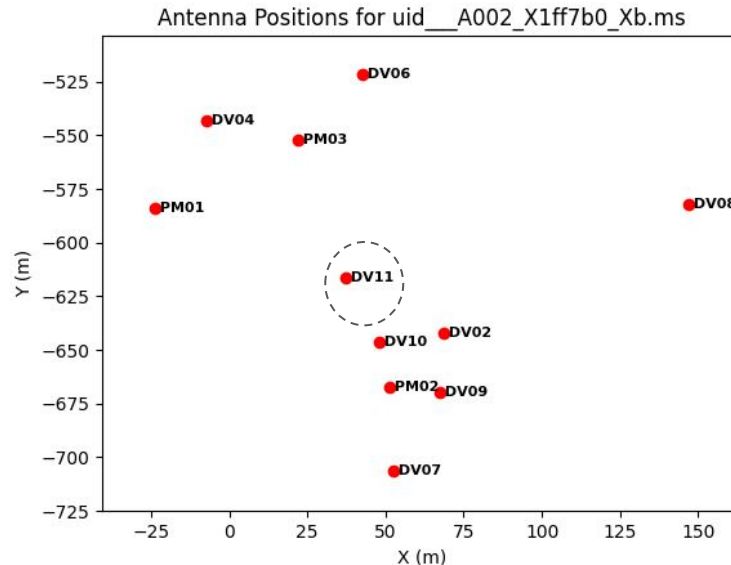
➤ Set the overall flux scale



Choosing a Reference Antenna

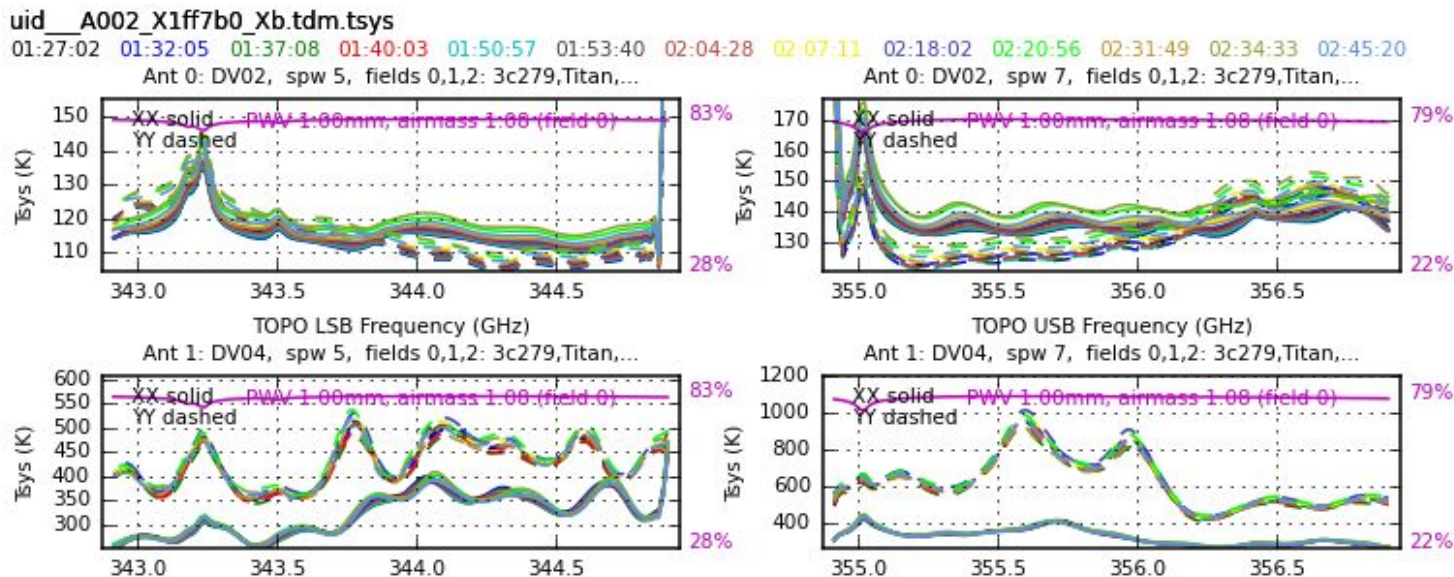
Gains calculated w.r.t. a reference antenna (DV11).

- Should be near the centre of the array, have good quality data over the observing run.
- Gains calculated for other antennas will be relative to this reference antenna (assume $\theta_{DV11}=0^\circ$)



Apply gain corrections from monitoring systems

- Variations in system temperature, water vapor monitoring (radio monitors), antenna pointing corrections.

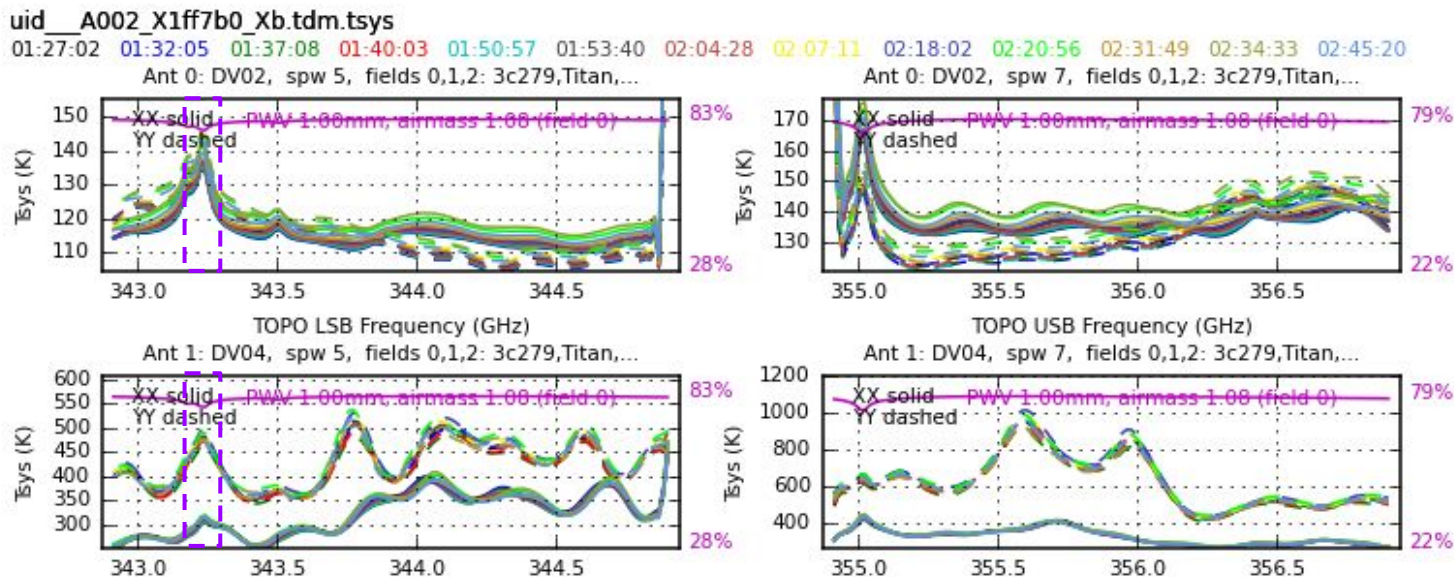


Lower Side Band

Upper Side Band

Apply gain corrections from monitoring systems

- Variations in system temperature, water vapor monitoring (radio monitors), antenna pointing corrections.



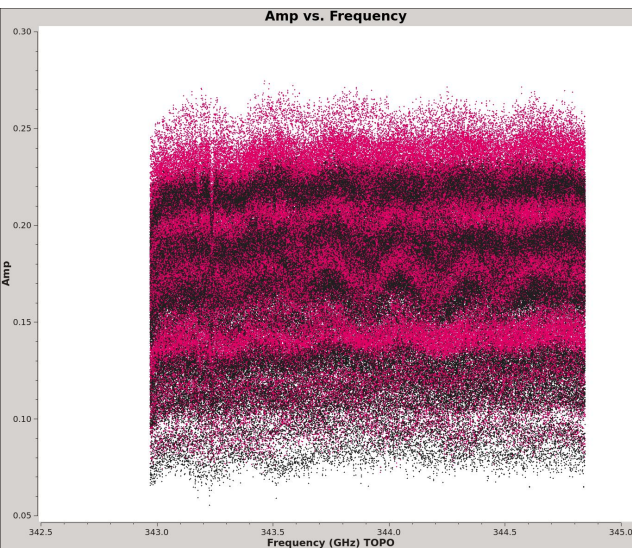
Lower Side Band

Upper Side Band

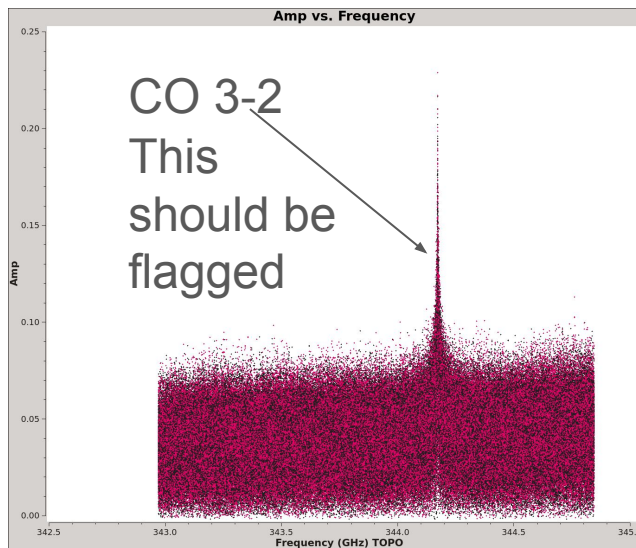
Note that we also have atmospheric lines, that correlate with high T_{sys} and should probably be flagged and removed.

LSB Amplitude vs Frequency for the different targets

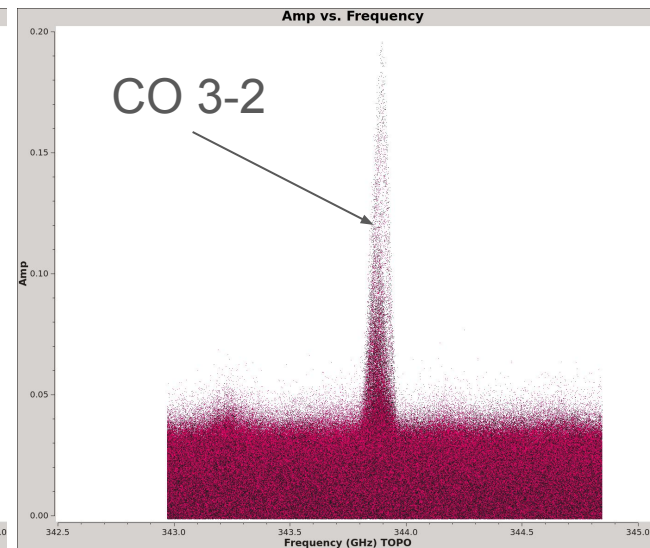
3C279



Titan

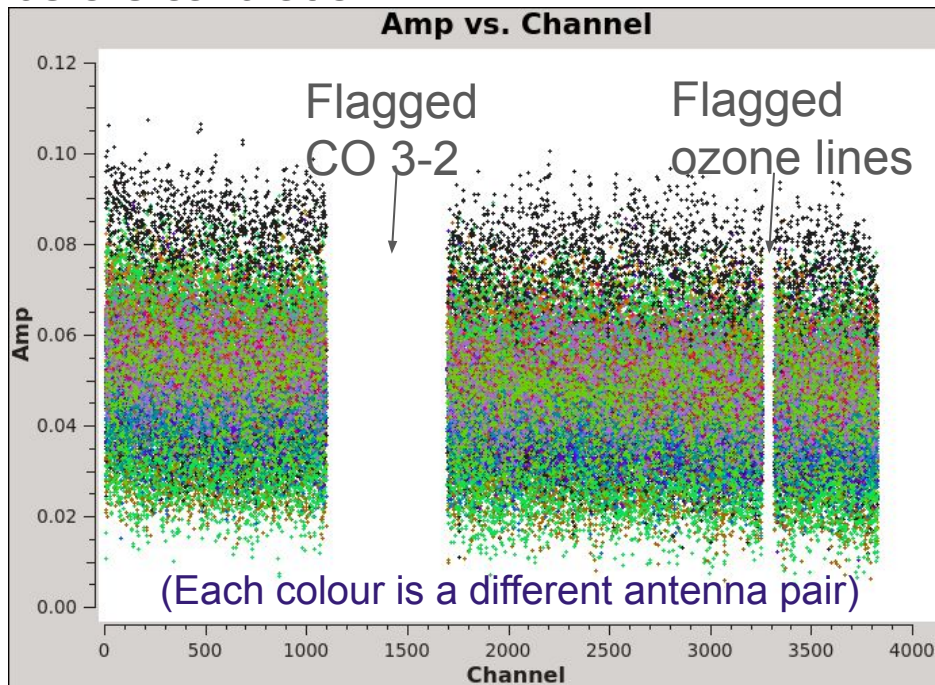


Antennae (Northern mosaic)

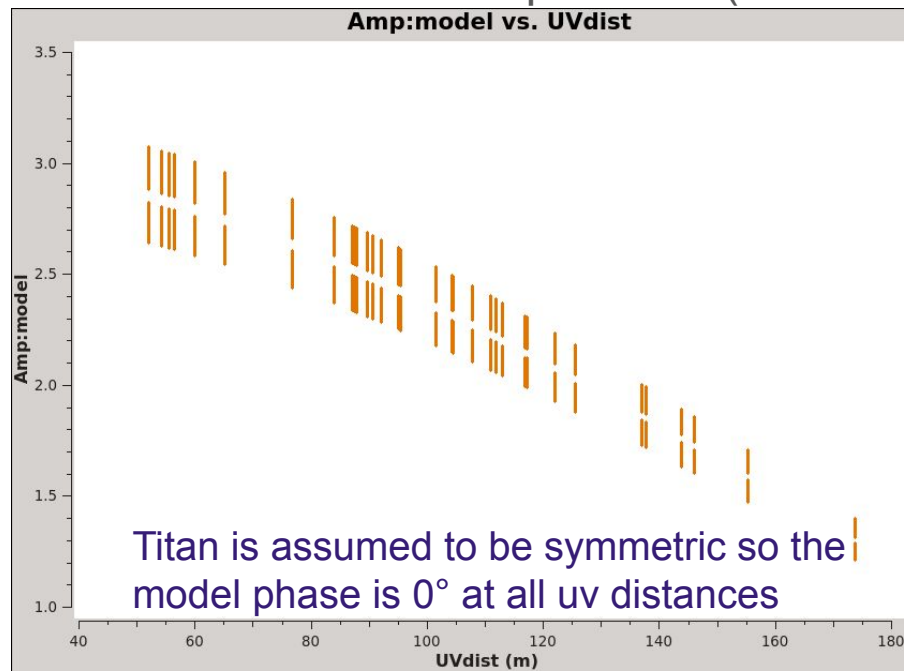


Flux Calibration: Titan

Amplitude vs Frequency channel for Titan before calibration



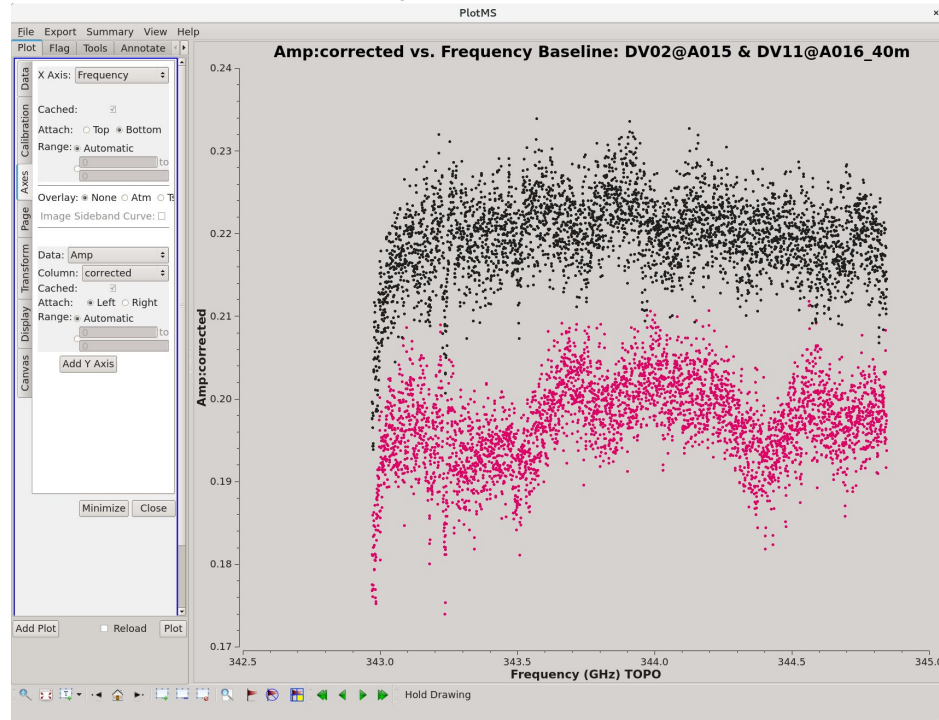
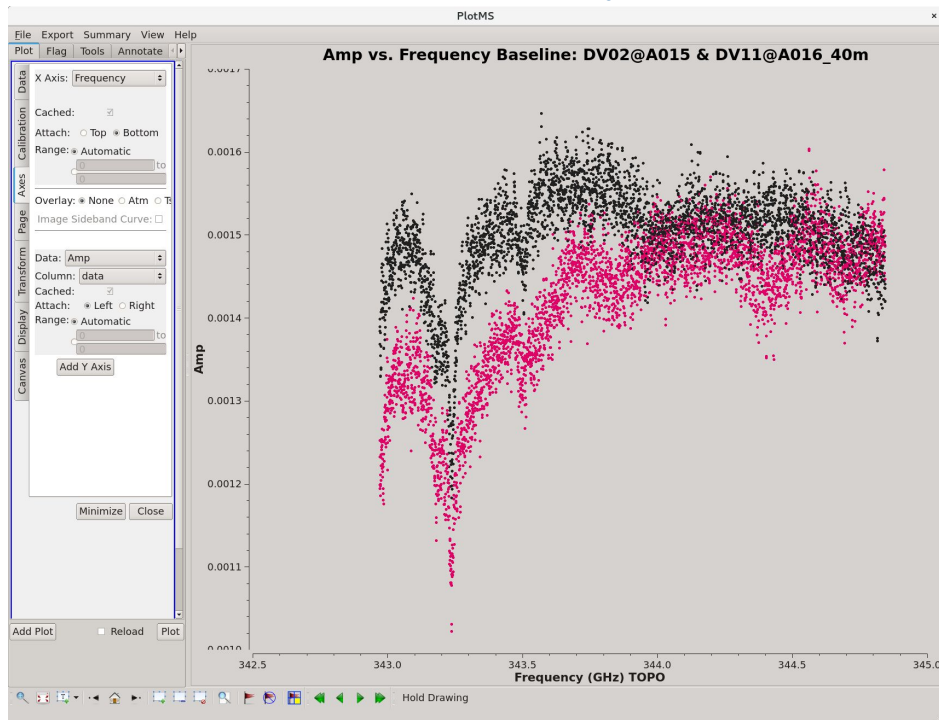
Model of Titan's visibility vs u-v distance for both autocorrelation products (XX & YY)



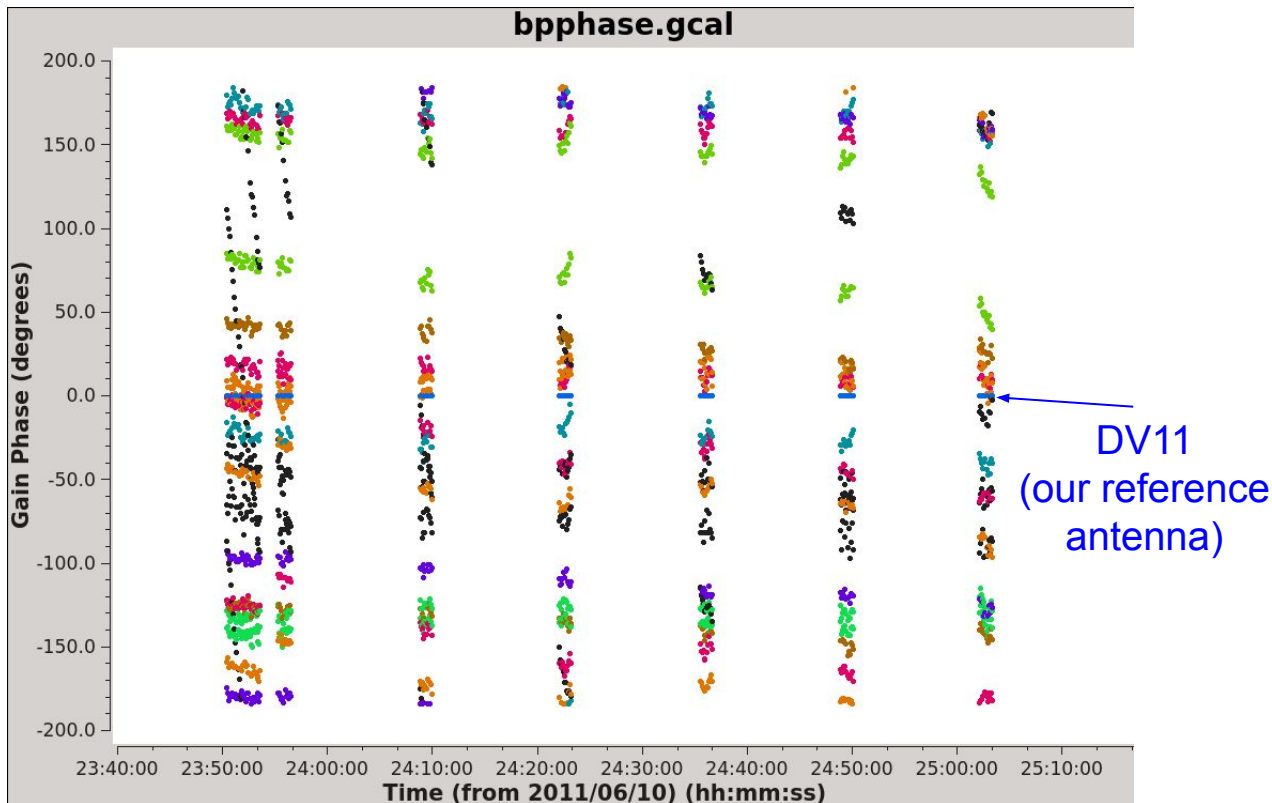
Band Pass observation:

Calibrate the gain response as a function of frequency $G(\nu)$

Band Pass Amplitude before Tsys, wvr corrections: Amplitude after Tsys, wvr corrections:



Track gain changes vs time

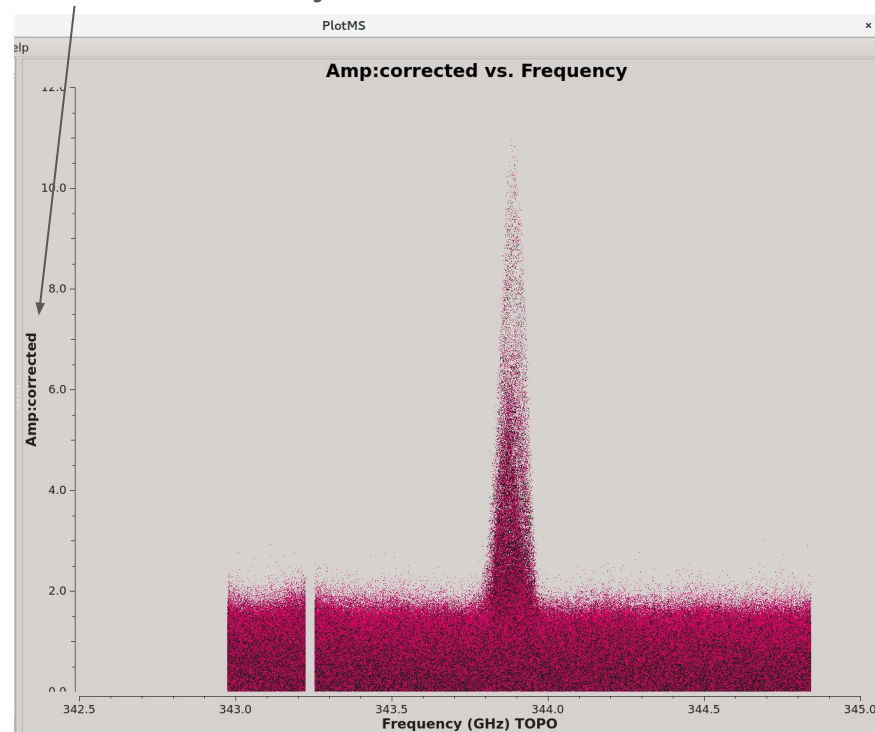
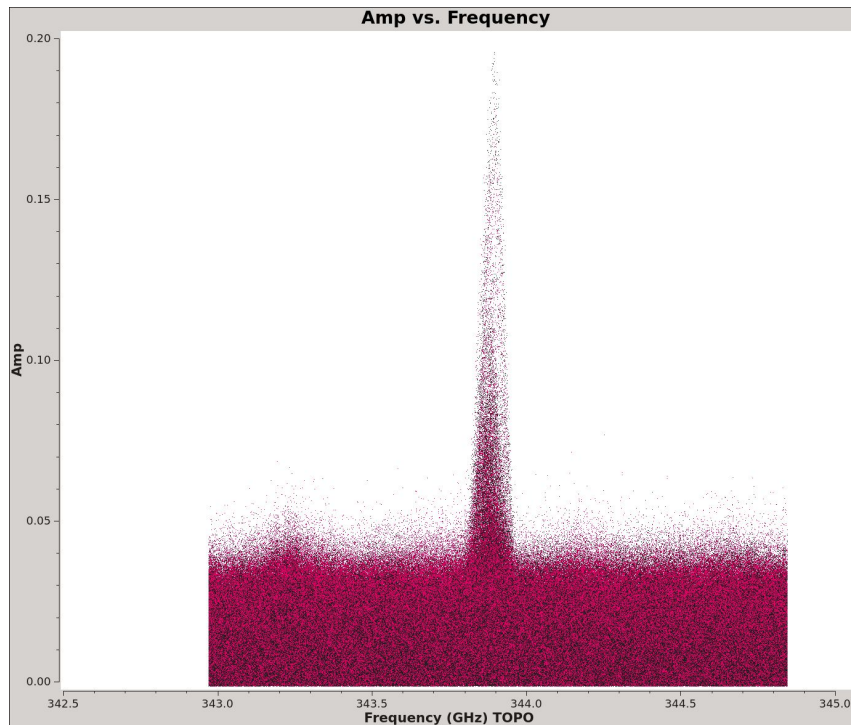


In between the calibrator observations we interpolate the phase solutions.

OR for a bright science target you could also attempt [self-calibration](#) on the science target observations.

Antennae Example: Before vs After Calibration

Units are now Jy



Closure Quantities:

Sometimes it is difficult to solve for the gain terms because they vary rapidly, or because there are few baselines, or the source is variable. In this case we can use **closure quantities**, which are insensitive to antenna gain terms.

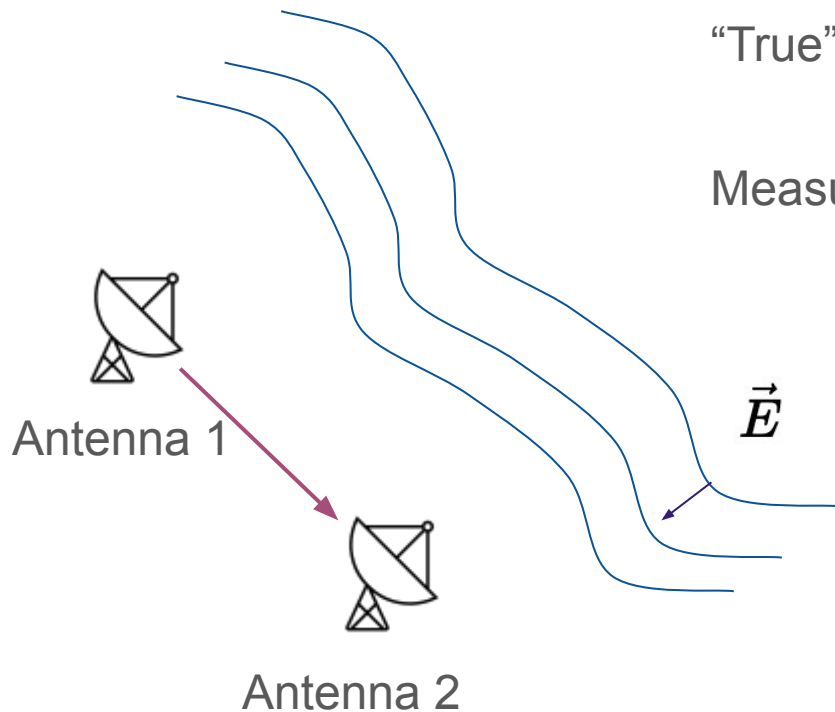
“True” visibility $V_{12} = \langle \mathbf{E}_1 \mathbf{E}_2^* \rangle = A_{12} e^{i\phi_{12}}$

Measured visibility $\hat{V}_{12} = \langle (G_1 \mathbf{E}_1)(G_2 \mathbf{E}_2)^* \rangle$

$$= G_1 \langle \mathbf{E}_1 \mathbf{E}_2^* \rangle G_2^*$$

$$= G_1 V_{12} G_2^* = G_{12} V_{12}$$

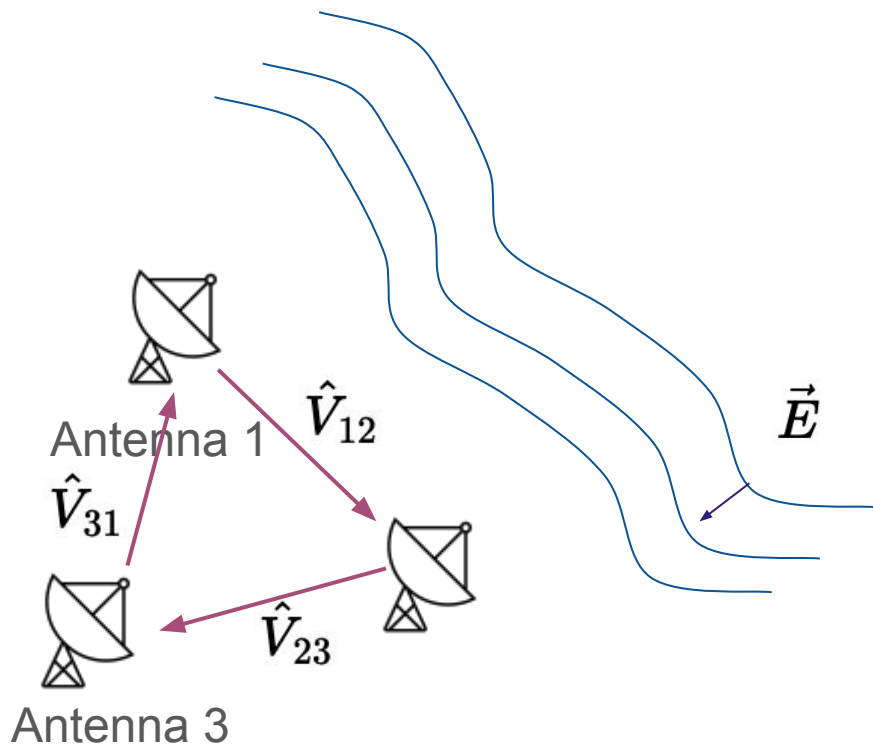
$$= g_1 g_2 e^{i(\theta_1 - \theta_2)} V_{12}$$



Closure Phase:

Any triangle of three antennas can be used to calculate a closure phase.

$$\hat{V}_{12}\hat{V}_{23}\hat{V}_{31} = (G_1 V_{12} G_2^*) (G_2 V_{23} G_3^*) (G_3 V_{31} G_1^*) = (G_1 G_1^*) (G_2 G_2^*) (G_3 G_3^*) V_{12} V_{23} V_{31}$$

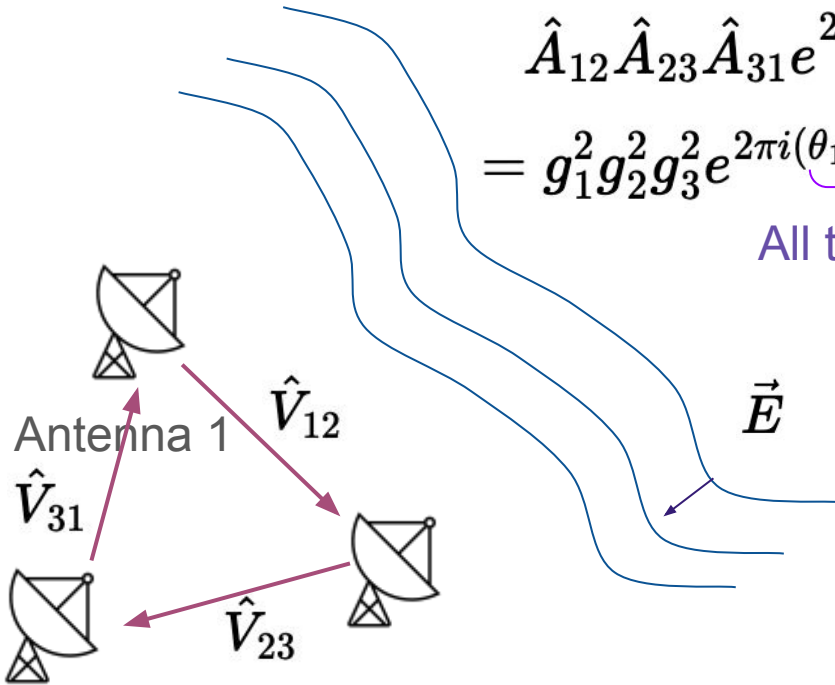


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$$\begin{aligned} & \hat{A}_{12} \hat{A}_{23} \hat{A}_{31} e^{2\pi i (\hat{\phi}_{12} + \hat{\phi}_{23} + \hat{\phi}_{31})} \\ &= g_1^2 g_2^2 g_3^2 e^{2\pi i (\underbrace{\theta_1 - \theta_1 + \theta_2 - \theta_2 + \theta_3 - \theta_3}_{\text{All the antenna phase gains cancel!}})} A_{12} A_{23} A_{31} e^{2\pi i (\phi_{12} + \phi_{23} + \phi_{31})} \end{aligned}$$



Antenna 3

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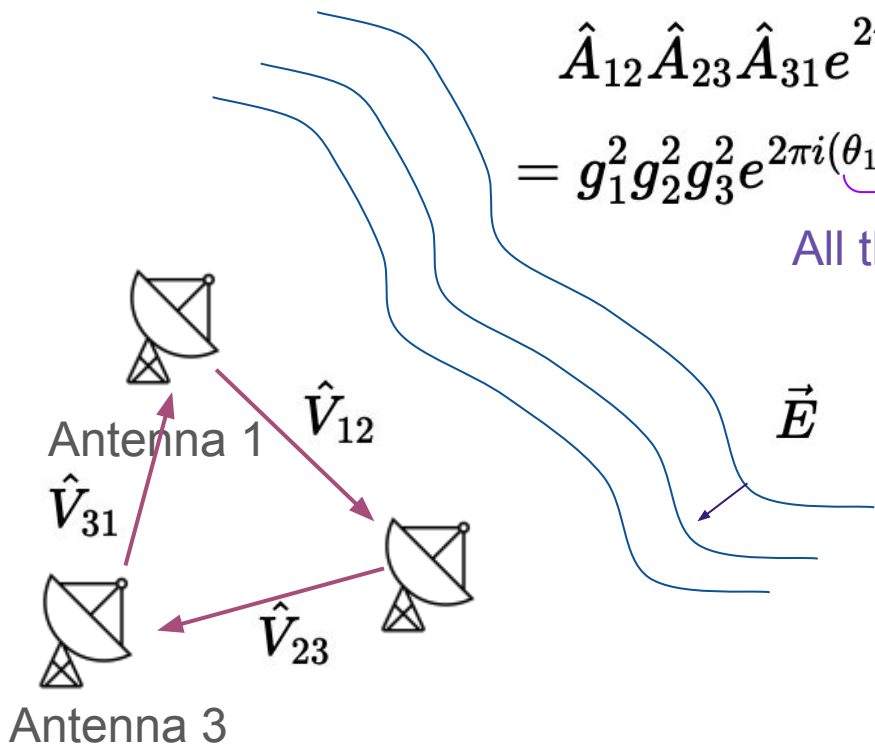
All the antenna phase gains cancel!

If we measure the phases of the visibilities we find:

$$\hat{\phi}_{12} + \hat{\phi}_{23} + \hat{\phi}_{31} = \phi_{12} + \phi_{23} + \phi_{31}$$

Closure phase

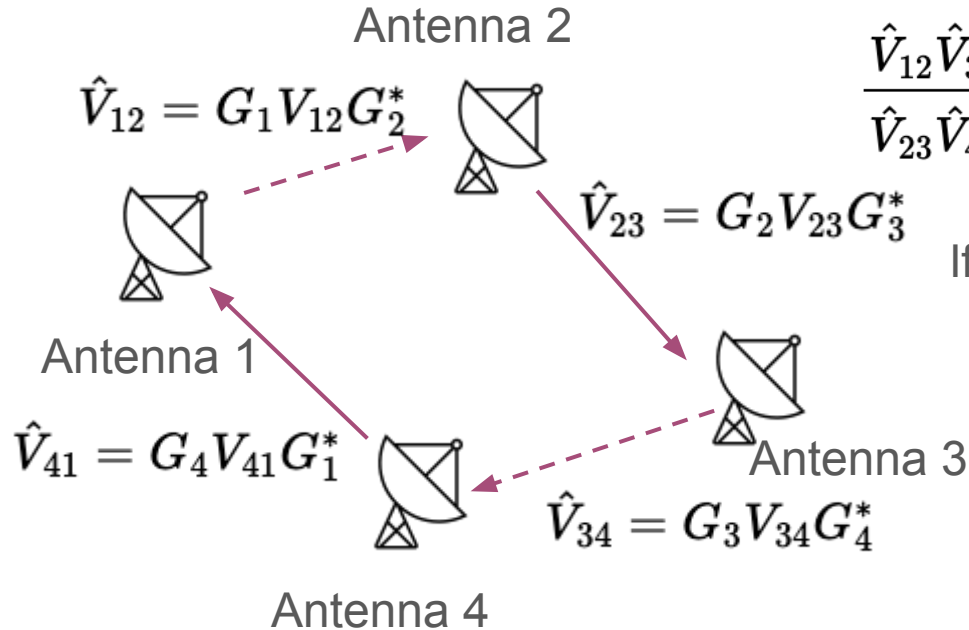
= 0 (+ noise) if the source is azimuthally symmetric



Closure Amplitude

Any closed quadrilateral of antennas can be use to define a closure amplitude relationship.

$$\frac{\hat{V}_{12}\hat{V}_{34}}{\hat{V}_{23}\hat{V}_{41}} = \frac{(G_1 V_{12} G_2^*) (G_3 V_{34} G_4^*)}{(G_2 V_{23} G_3^*) (G_4 V_{41} G_1^*)}$$



$$\frac{\hat{V}_{12}\hat{V}_{34}}{\hat{V}_{23}\hat{V}_{41}} = \left(\frac{G_1}{G_1^*}\right) \left(\frac{G_2^*}{G_2}\right) \left(\frac{G_3}{G_3^*}\right) \left(\frac{G_4^*}{G_4}\right) \frac{V_{12}V_{34}}{V_{23}V_{41}}$$

If we take the amplitude of both sides we find:

$$\left| \frac{\hat{V}_{12}\hat{V}_{34}}{\hat{V}_{23}\hat{V}_{41}} \right| = \left| \frac{V_{12}V_{34}}{V_{23}V_{41}} \right|$$

Using closure quantities:

Whatever solutions that you get from your calibration the measured source amplitudes and phases **must satisfy the closure relations!**

Phase:

$$\hat{\phi}_{mn} + \hat{\phi}_{np} + \hat{\phi}_{pm} = \phi_{mn} + \phi_{np} + \phi_{pm}$$

Number of independent closure phases:

$$\frac{(N_{ant} - 1)(N_{ant} - 2)}{2}$$

For $N_{ant} = 10$ this is 36 closure phases

Amplitude:

$$\left| \frac{\hat{V}_{mn} \hat{V}_{pq}}{\hat{V}_{np} \hat{V}_{qm}} \right| = \left| \frac{V_{mn} V_{pq}}{V_{np} V_{qm}} \right|$$

$$\frac{1}{2} N_{ant} (N_{ant} - 3)$$

For $N_{ant} = 10$ this is 35 closure amplitudes

So now we have calibrated visibilities!

Next we could go straight to making an image by taking the inverse Fourier transform of our visibility data

... or we could just fit a parameterized model to the visibility data directly (forward modeling).

Question for the group: Why would we want to fit models to the u,v data rather than going straight to imaging?

Example analytic model fit:

2-D Gaussian:

- Flux density S_0
- Location (x_0, y_0)
- Width (σ)

$$V(u, v) = S_0 e^{-2\pi i(ux_0 + vy_0)} e^{-2\pi^2 \sigma^2 (u^2 + v^2)}$$

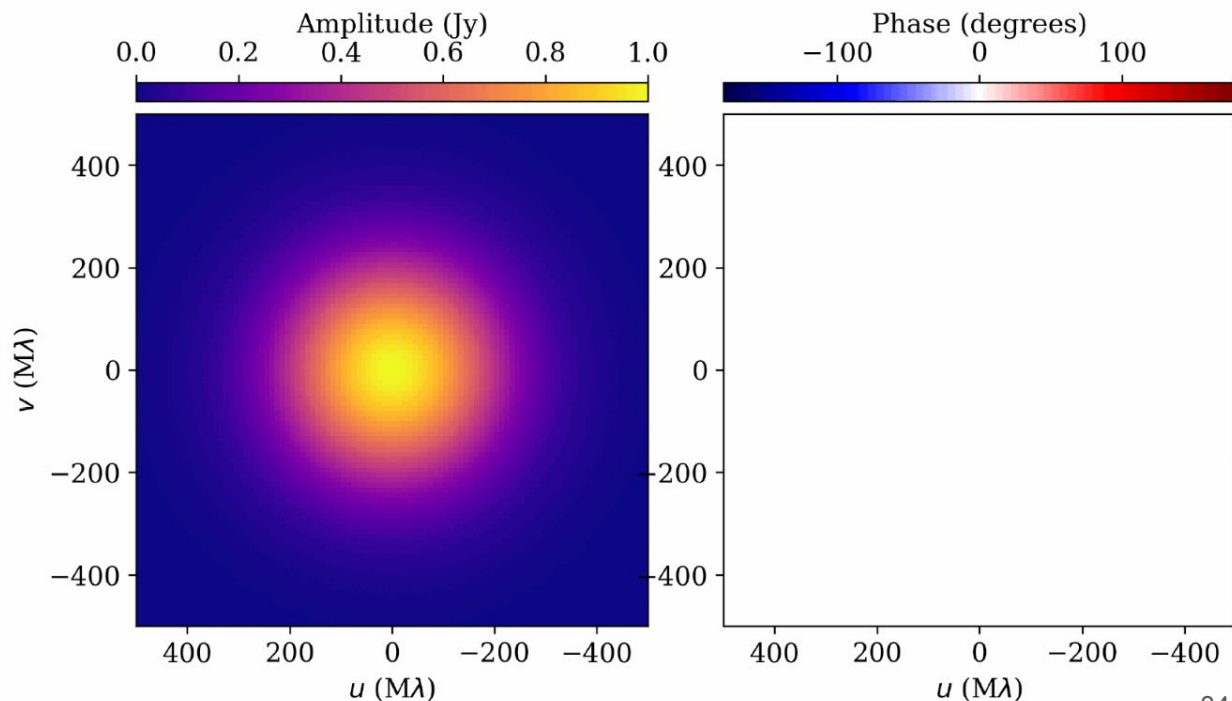
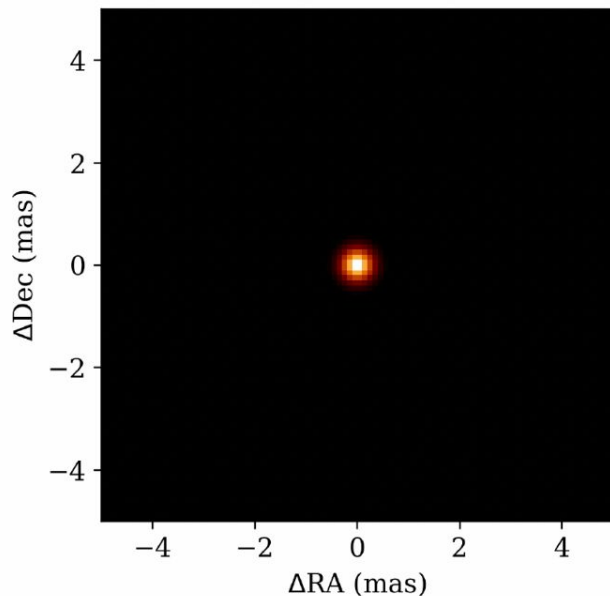


Image credit: Dominic Pesce

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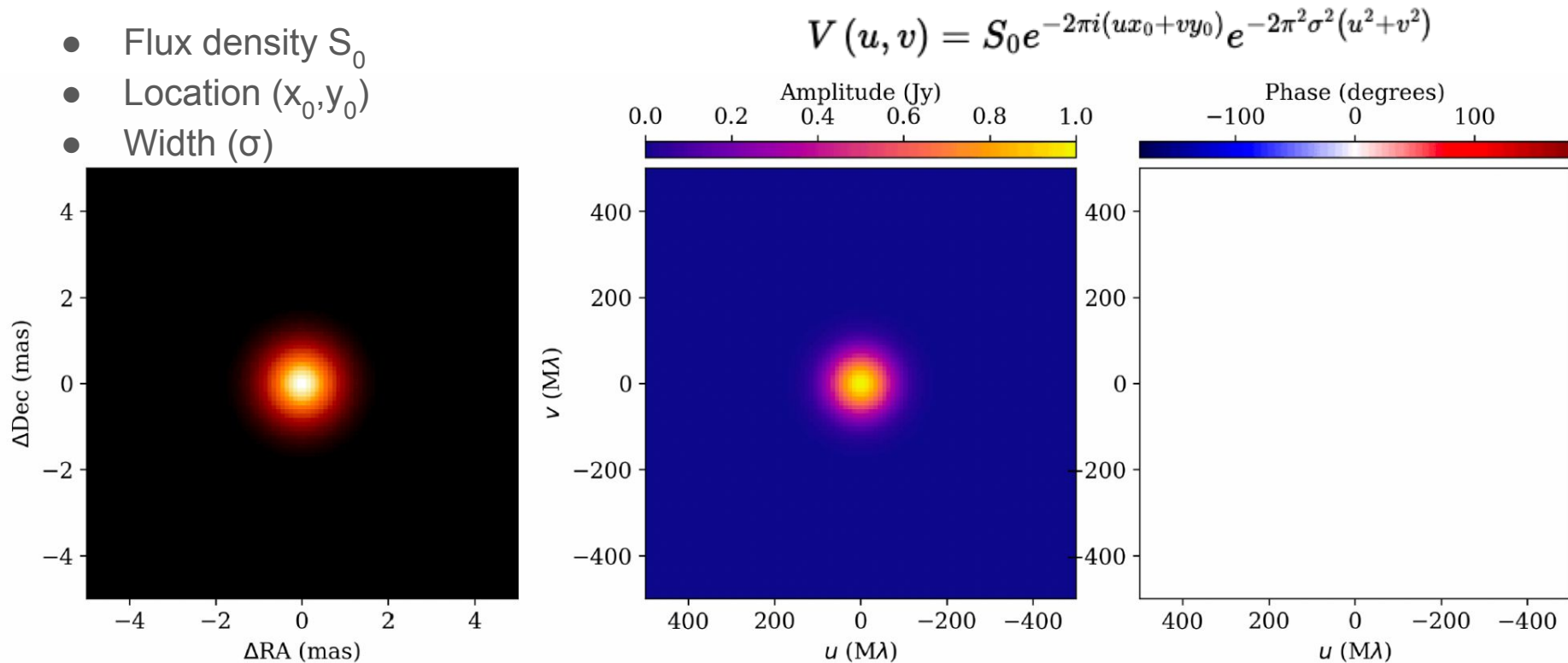


Image credit: Dominic Pesce

Other simple analytic models:

ring

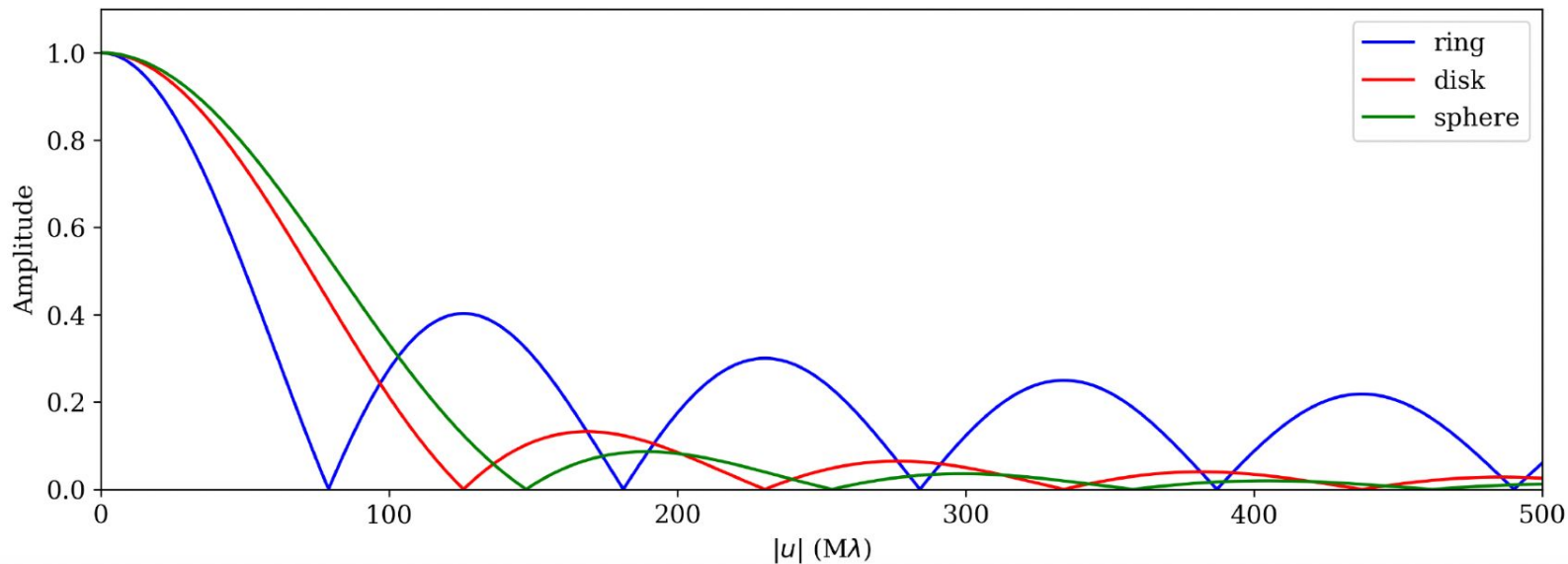
$$V(u) = S_0 J_0(2\pi R u)$$

disk

$$V(u) = \frac{S_0}{\pi R u} J_1(2\pi R u)$$

sphere

$$V(u) = \frac{3S_0}{4\pi(Ru)^{3/2}} J_{3/2}(2\pi R u)$$



Fitting to visibilities

Normally the strategy is to minimize a χ^2 or if using Bayesian forward modelling maximize a likelihood.

$$\chi^2 = \sum_{i=1}^{n_d} \frac{[V(u_i, v_i) - V_m(u_i, v_i, \vec{p})]^2}{\sigma_i^2}$$

Measured visibility

Visibility model for parameters $\{p_i\}$

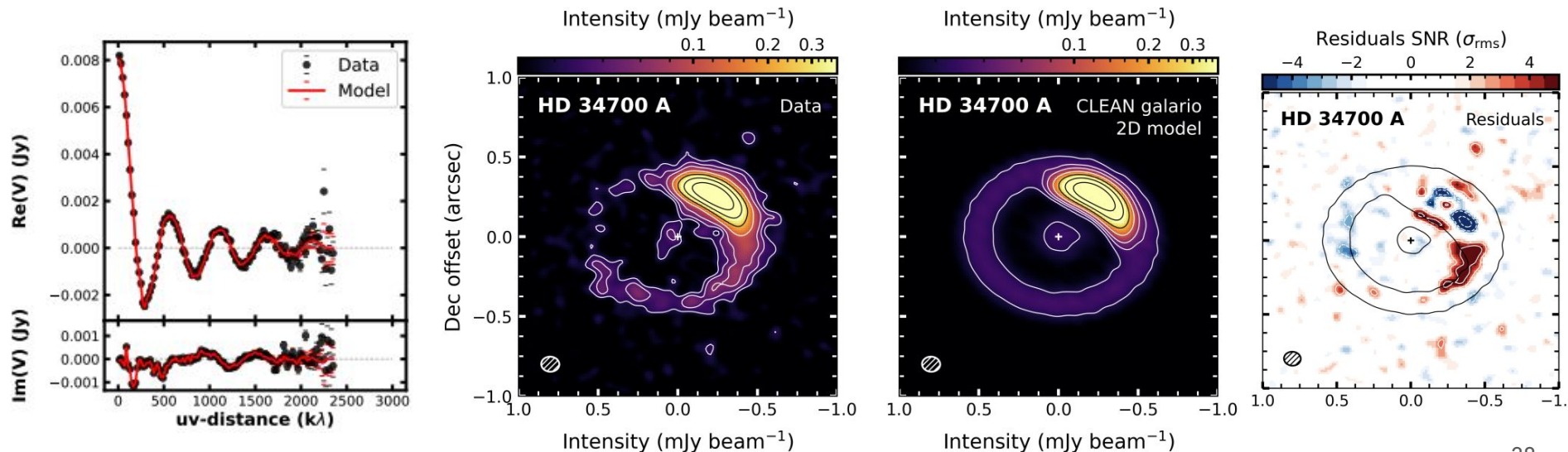
For a perfect fit χ^2 is expected to be $n_d - n_p$, or the reduced chi-squared $\chi^2/(n_d - n_p)=1$

Example #1: Fitting asymmetric dust structure in protoplanetary disk around Herbig Ae star 34700A with high resolution (0.1'') ALMA Band 6 continuum data

Uses visibility fitting code [galarío](#) which performs Bayesian fitting and uses Monte Carlo Markov chains to explore the parameter space.

$$I_{\text{HD 34700 A}}(r, \theta) = G(f_0, \sigma_0) + GR(f_1, r_1, \sigma_{r,1}) + GAa(f_1^a, r_1^a, \sigma_{r,1}^a, \theta_1^a, \sigma_{\theta,11}^a, \sigma_{\theta,1}^a),$$

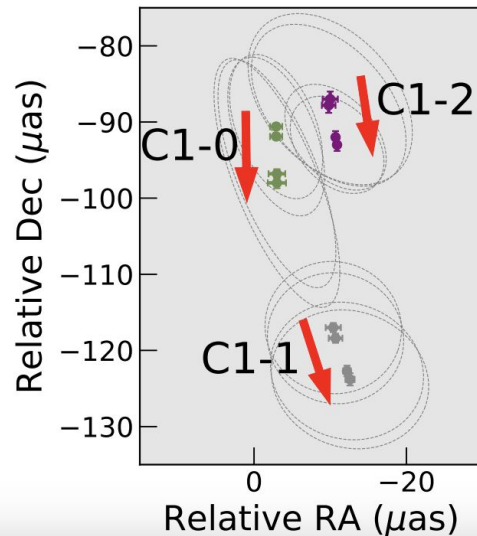
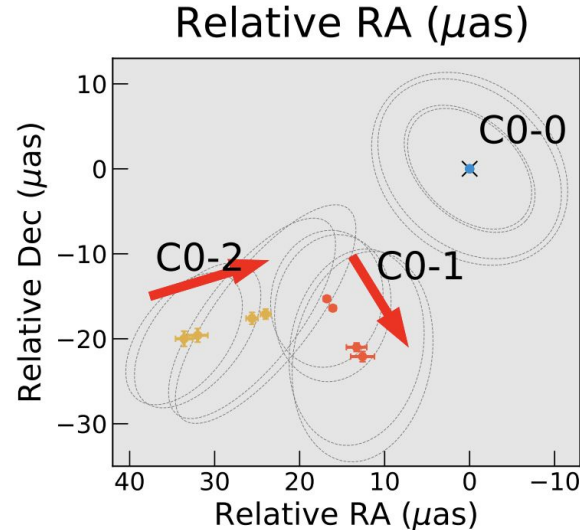
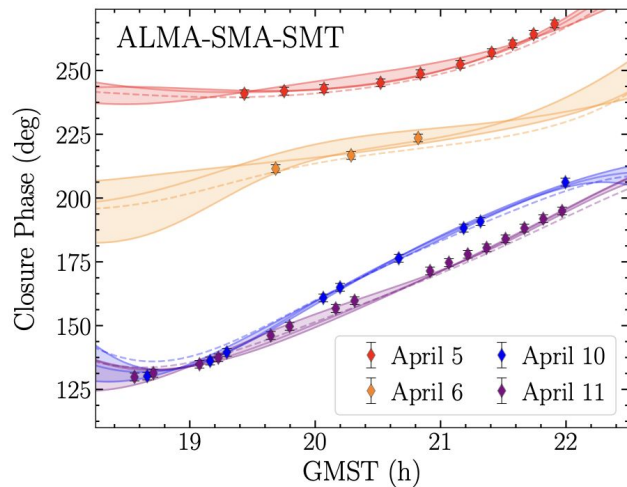
Gaussian Gaussian symmetric ring Gaussian asymmetric ring



Example #2: [Kim et al. 2020](#)

230 GHz Event Horizon Telescope Observations of the inner jet from blazar 3C279 with 20 μ arcsecond (0.13pc) resolution over days.

Closure phases show changes from day to day. They modelling the amplitudes and closure phases over time to track 6 different Gaussian components associated with the jet components.



Creating Images from Visibility Data

Always keep in mind: an image is a model of your data.

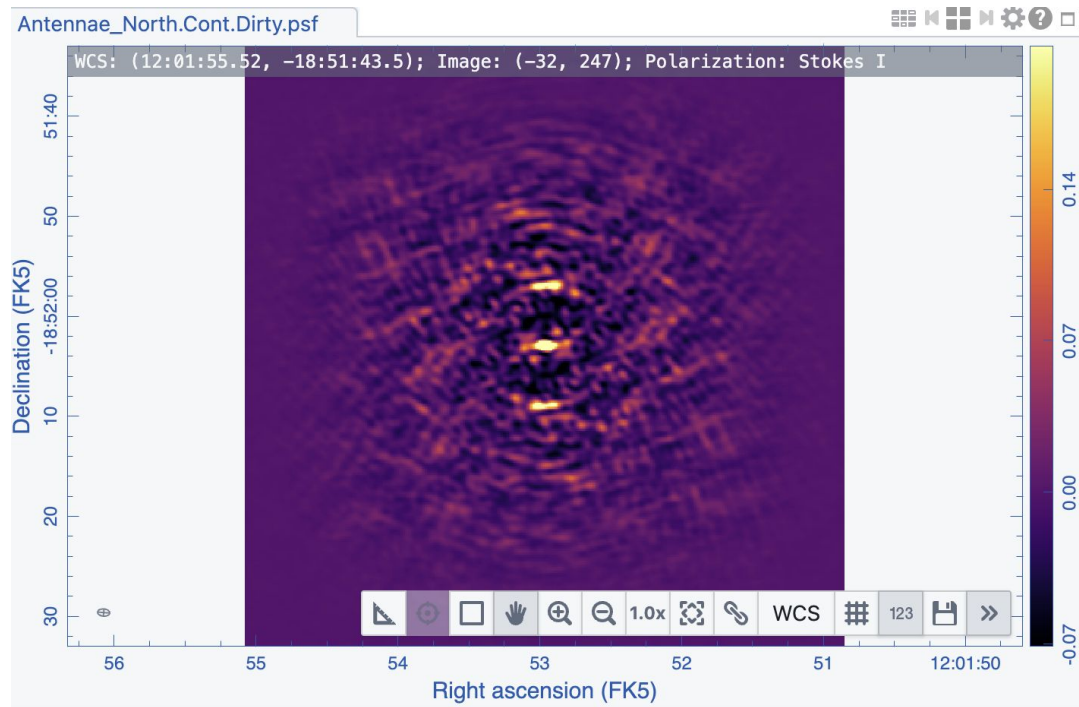
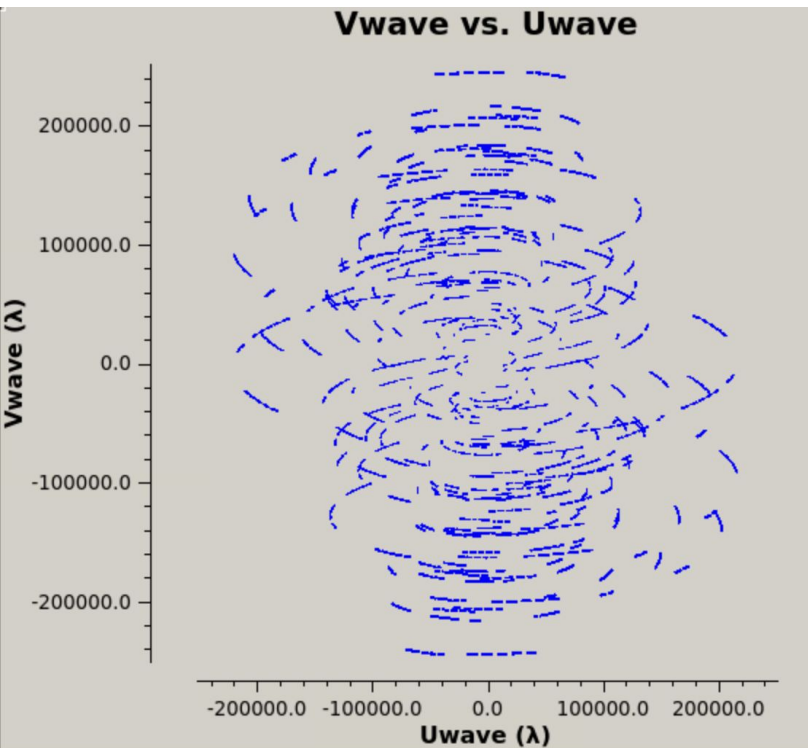
$$I(l, m) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} V(u, v) e^{2\pi i(ul+vm)} du dv$$

The problem: we have only measured the visibilities at a discrete number of u, v distances:

Transfer function Visibility weights e.g. $w(u, v) = \frac{1}{\sigma_{V_{uv}}^2}$

$$V_{meas}(u, v) = \underbrace{W(u, v)w(u, v)}_{\substack{\text{Fourier} \\ \text{transform of the} \\ \text{"dirty" beam } b_0}} V(u, v)$$

“Dirty” Beam for the ALMA Antennae Band 7 continuum data



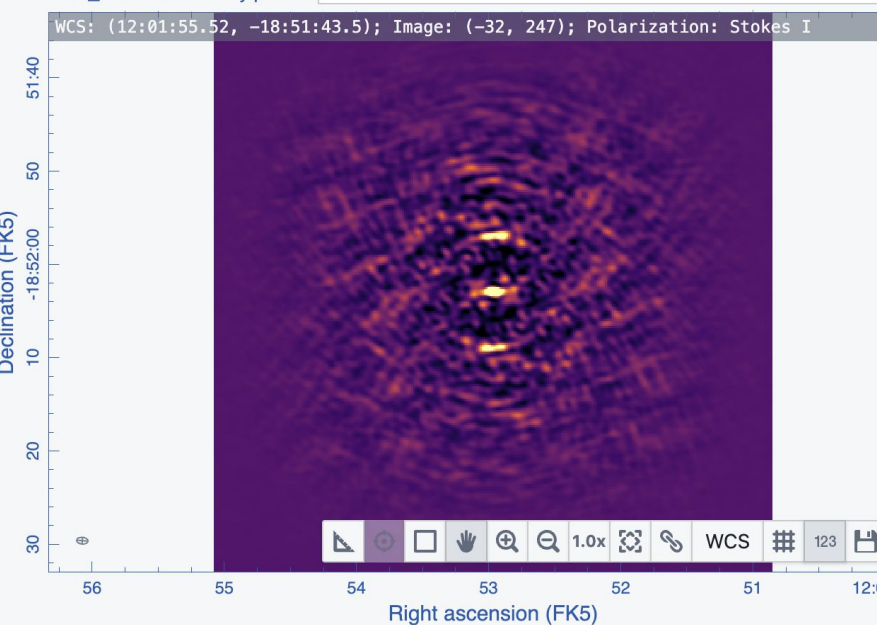
“Dirty” Image for the ALMA Antennae Band 7 continuum data

$$V_{meas}(u, v) = W(u, v)w(u, v)V(u, v)$$

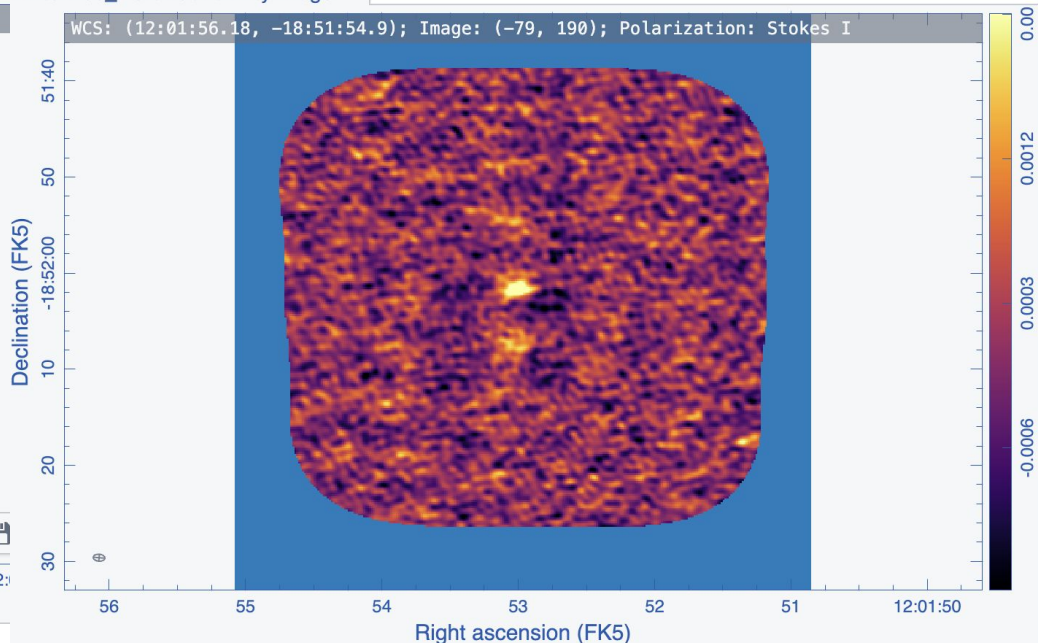
Transfer function is 0 everywhere except
where we have measurements

$w = 1/(\sigma_{uv})^2$ (natural weighting)

Antennae_North.Cont.Dirty.psf



Antennae_North.Cont.Dirty.image



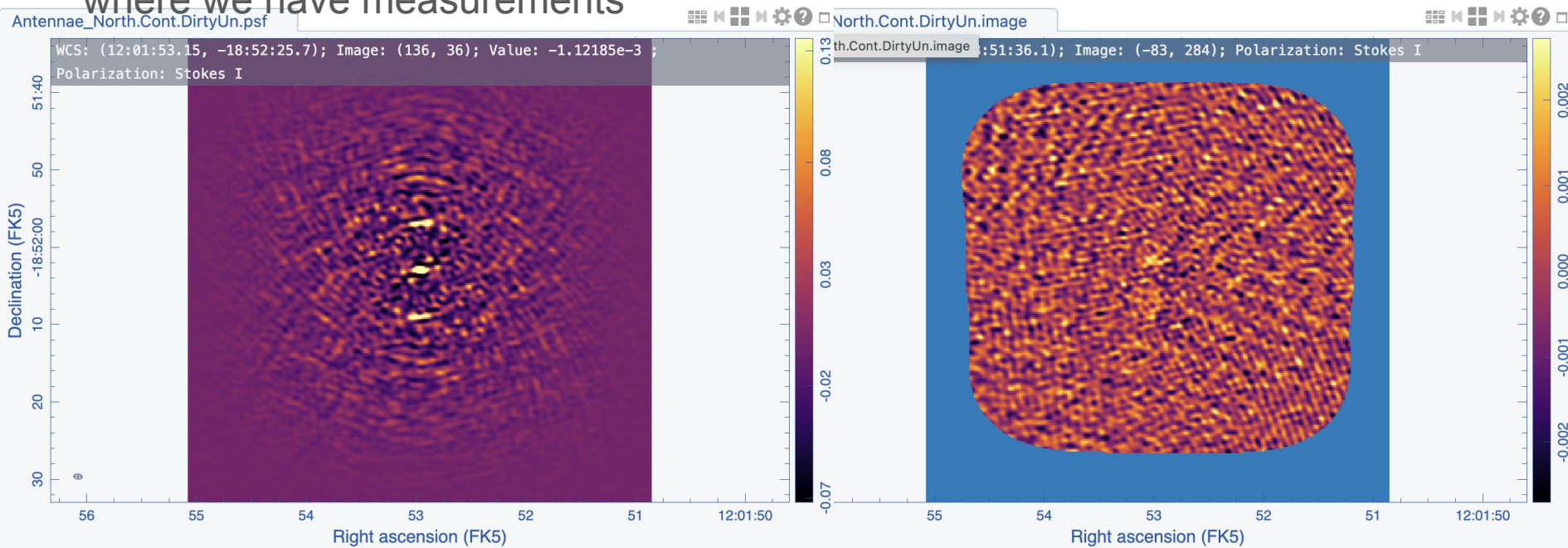
$$(\text{Dirty Image}) = (\text{True image}) \otimes (\text{Dirty Beam})$$

“Dirty” Image for the ALMA Antennae Band 7 continuum data

$$V_{meas}(u, v) = W(u, v)w(u, v)V(u, v)$$

Transfer function is 0 everywhere except
where we have measurements

$$w = 1/(\rho_{uv} \sigma_{uv})^2 \text{ (uniform weighting)}$$



$$(\text{Dirty Image}) = (\text{True image}) \otimes (\text{Dirty Beam})$$

Deconvolution with CLEAN

Getting a better quality image relies on modelling the visibilities for unmeasured u,v values. There are lots of methods of doing this, but for the sake of time I'm going to focus on the most common, the CLEAN algorithm ([Hogbom, 1974](#))

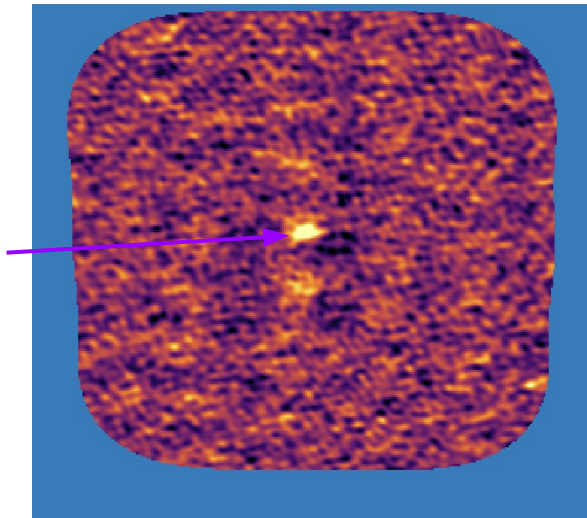
Basic idea: $I(l,m)$ is composed of a series of point sources.

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Step 1:
Take your
dirty map and
identify the
brightest
structure.

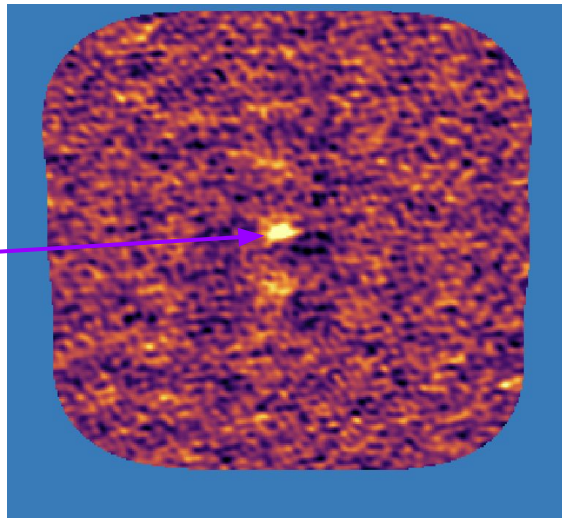


Deconvolution with CLEAN

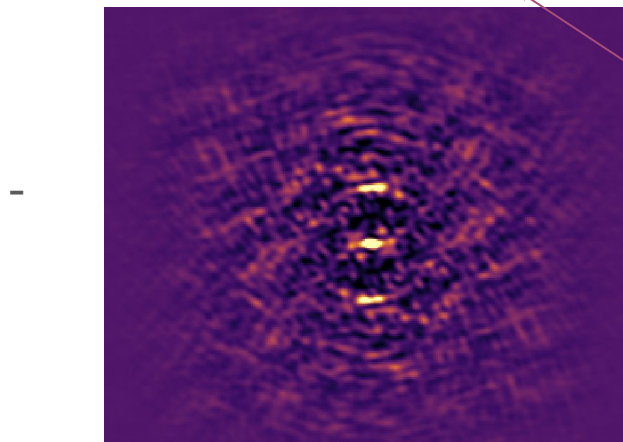
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Convolve a point source centered on the peak
with amplitude $A \times$ fraction g and subtract



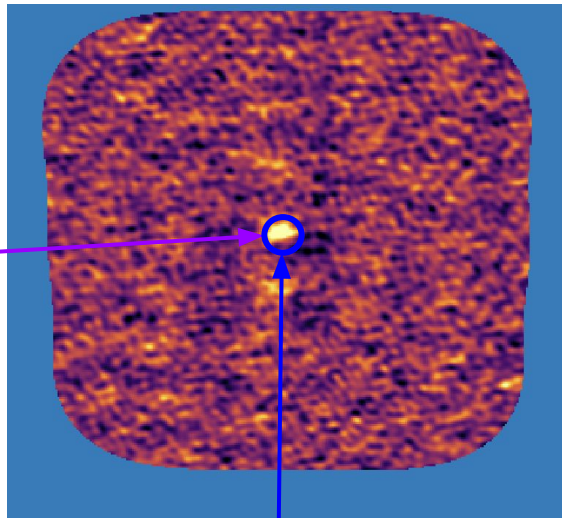
Loop gain
($g=0.1$ is
standard)

Deconvolution with CLEAN

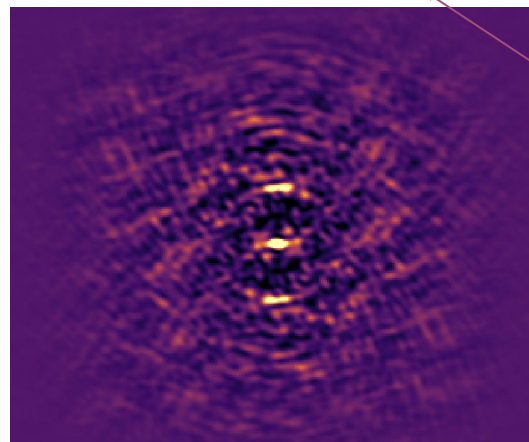
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($g=0.1$ is
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In some cases we may want to define a mask to tell clean where clean is allowed to remove point sources. Defining a mask can be very complicated for maps with a lot of extended structure!

Deconvolution with CLEAN (2)

Step 2:

Add a point source
of the same
amplitude to your
CLEAN model



Deconvolution with CLEAN (2)

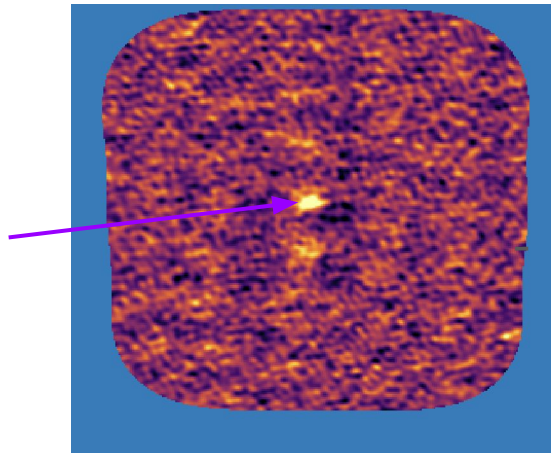
Step 2:

Add a point source of the same amplitude to your CLEAN model

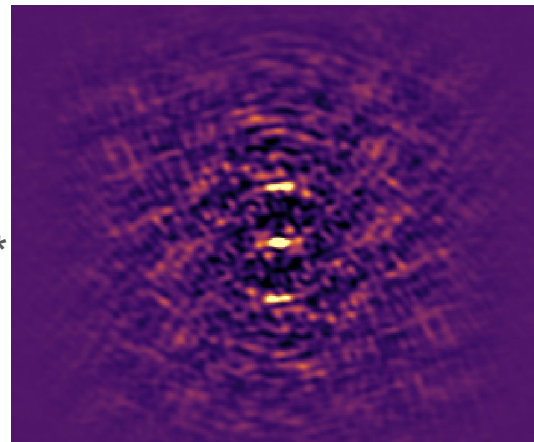


Step 3:

Take the residual dirty map from and identify the brightest structure, and subtract another point source convolved with the dirty beam.



$-g^*A^*$



Deconvolution with CLEAN (2)

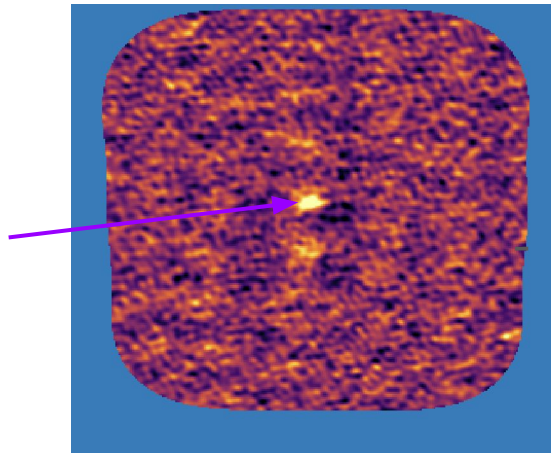
Step 2:

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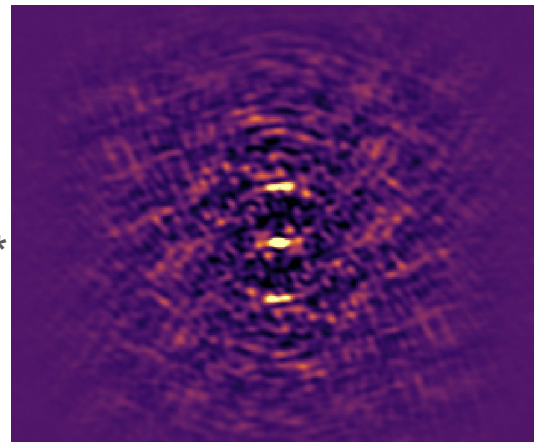


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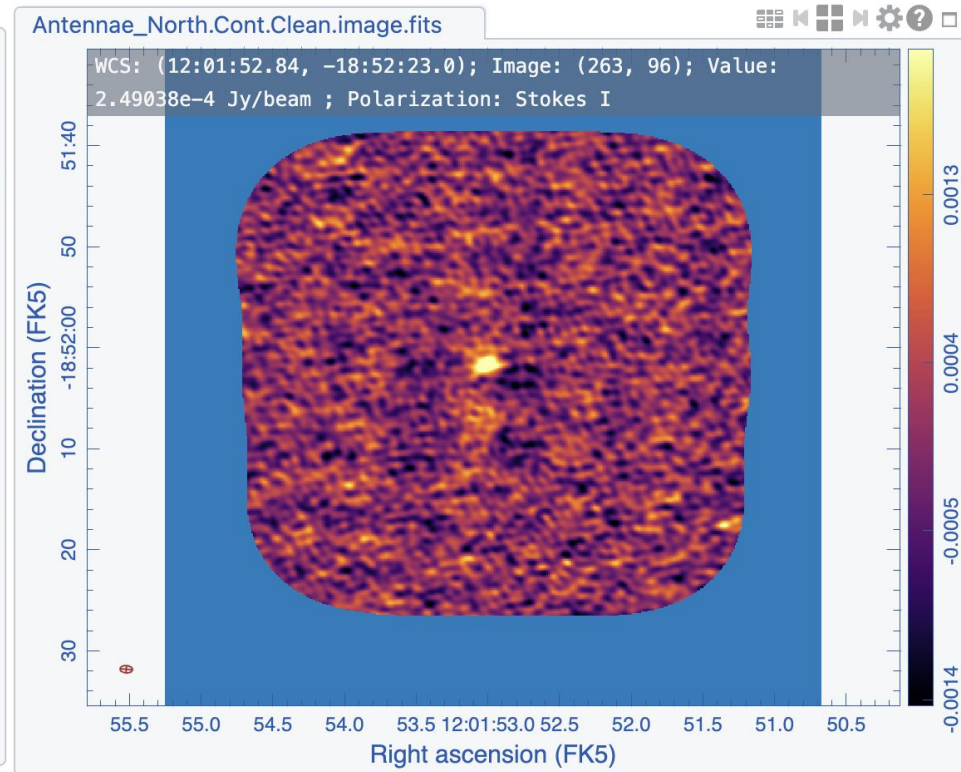
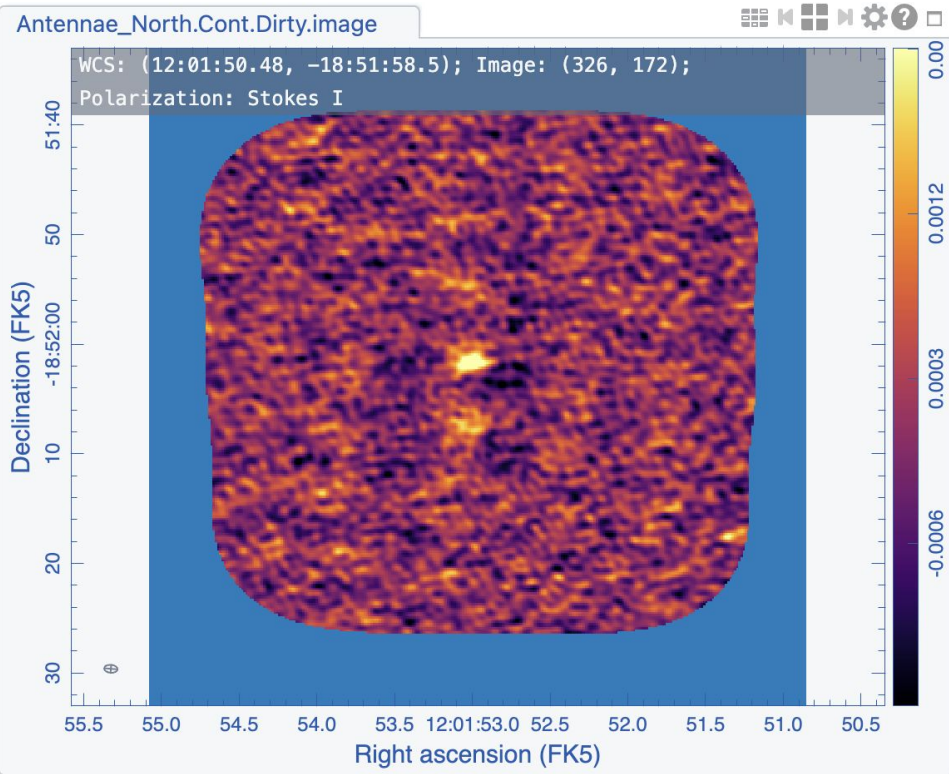


$-g^*A^*$



Repeat steps 2 and 3 until you have removed all structures down to I_{thresh} .

CLEAN map:



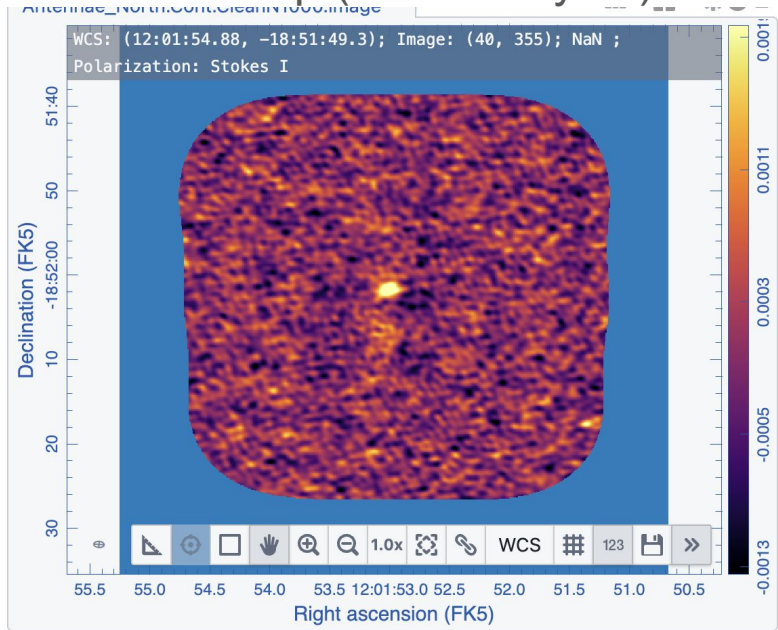
CLEAN maps are the model convolved by a clean beam (sidelobes suppressed) of the same size + the residual map.

A word of caution: CLEAN should be supervised!

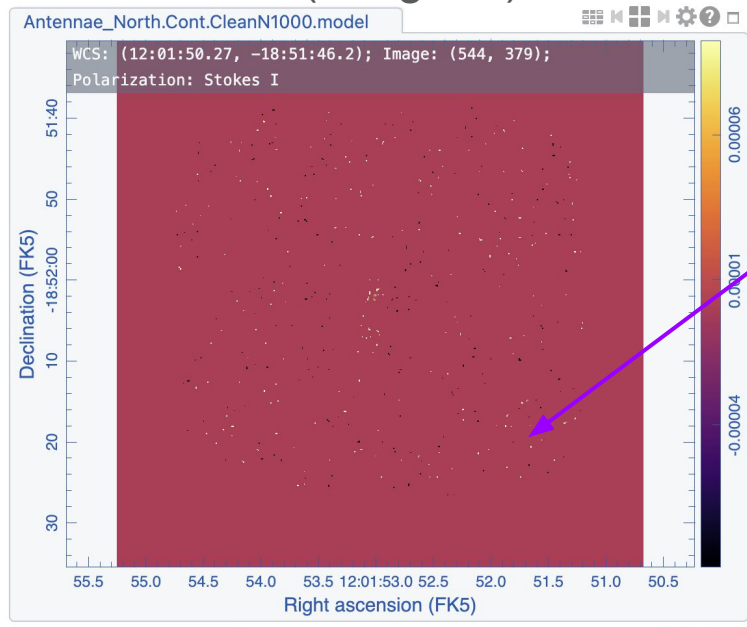
Often you will stop and check CLEAN after a certain number of iterations. If you leave it to CLEAN down to the map noise rms like I did in the example below it may try to subtract noise and add it to your model.

Example: casa tclean, 1000 iterations, threshold 0.4mJy (rms map noise)

Map (looks okayish)



Model (not good)



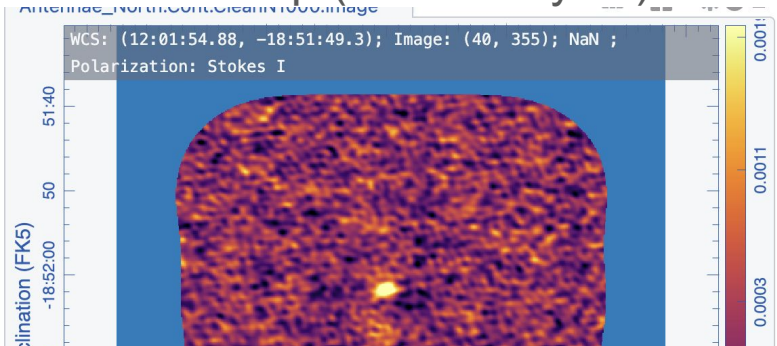
I've
cleaned
too deep
and added
noise to
my clean
model

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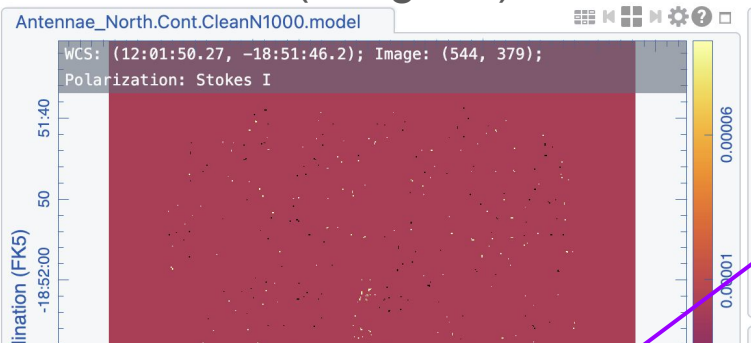
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Possible Solutions:

- Set a higher threshold (don't clean too deep)
- Use a mask to tell tclean where it can remove flux
- Check on tclean after every 50 iterations

Summary

Interferometric observations require careful calibration to recover complex visibilities.

There are a variety of measurables that can be investigated to check the quality of your data set, such as baseline amplitudes and phase measurement, but also antenna gain invariant quantities like closure phases and amplitudes.

Forward modelling of the visibility data is a powerful (though computationally intensive) tool for studying your source.

Deconvolution imaging techniques like CLEAN can be used to create maps from the source data, but remember that these algorithms are also creating a model of the source intensity.