The Basics of Interferometry

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Who am I?

Born in small town USA

Very little science in my high school

Undergraduate at small, 2000 person school in MN → introduction to radio astronomy

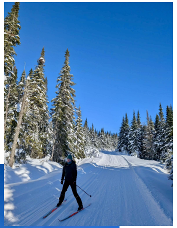
Serious concussion → left school semester, reduced course load

Internship studying galaxies & their jets

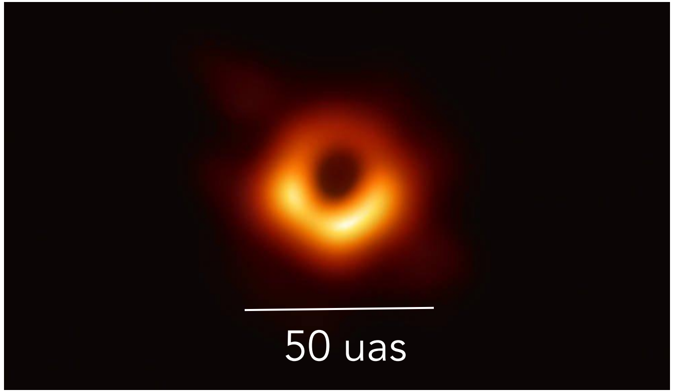
MSc + PhD at McGill

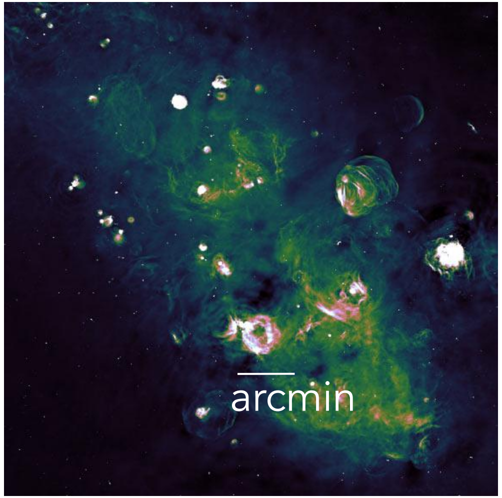
Postdoc at McGill

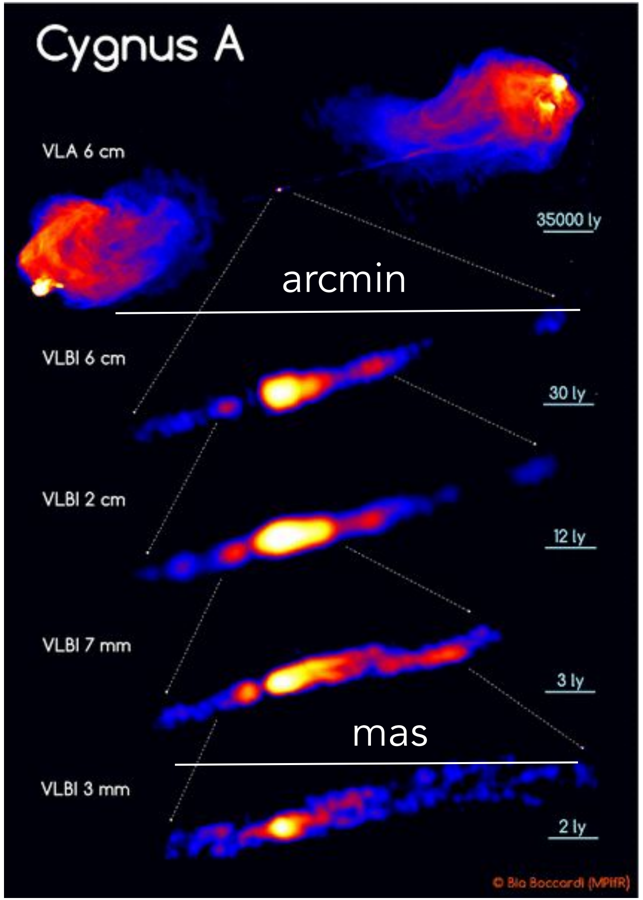
Canadian SKA fellow @ McGill & one of the CHIME/FRB operations leads

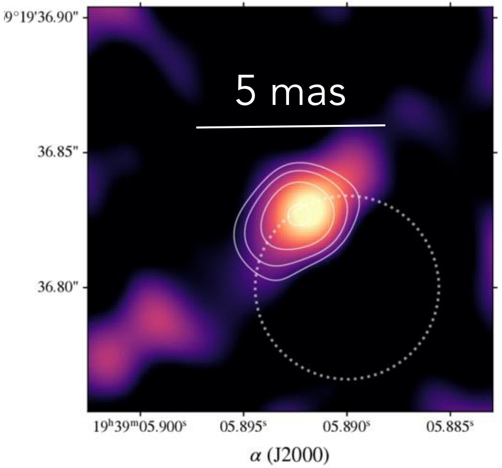


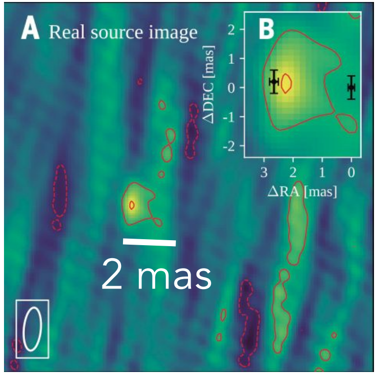
The night sky exists on many different scales











How do we sample this exciting night sky?

λ/D – sets our telescope resolution

To build FAST (500-m dish), $250 million CAD

- Arcminute resolution at 1.4 GHz

How do we do better? Large N at large distances

- large effective collecting area
- Long baselines

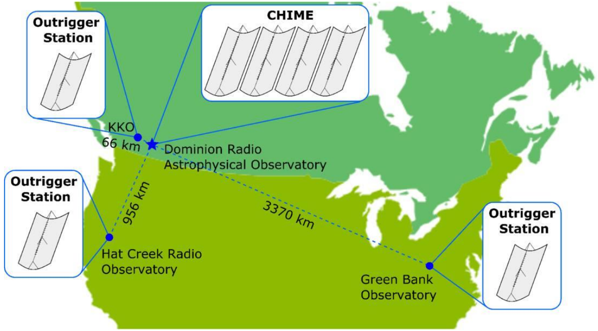
Interferometry

Interferometry: everywhere all the time

CHIME, British Columbia

LOFAR, Netherlands

CHIME/Outriggers, North America



1° 1″ sub-mas

SKA, Australia & South Africa

VLA, United States

VLBA, United States

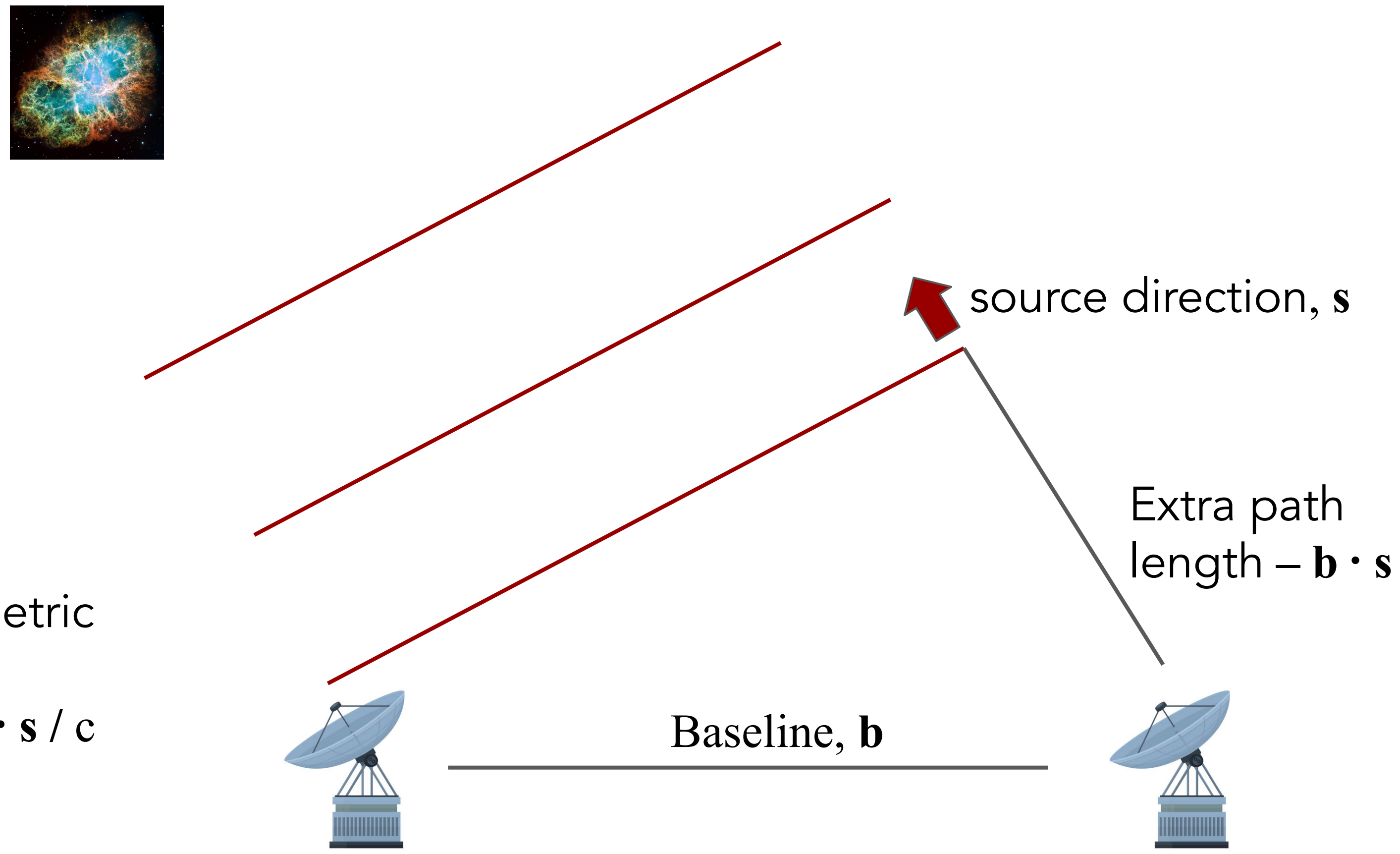
Goals for the next 45-minutes

- Gain intuition for the most basic interferometer
  - How can we map sky brightness back to measured electric fields?
- Introduction to the UV-plane
- Relaxing some of the basic interferometer assumptions – what problems pop up?

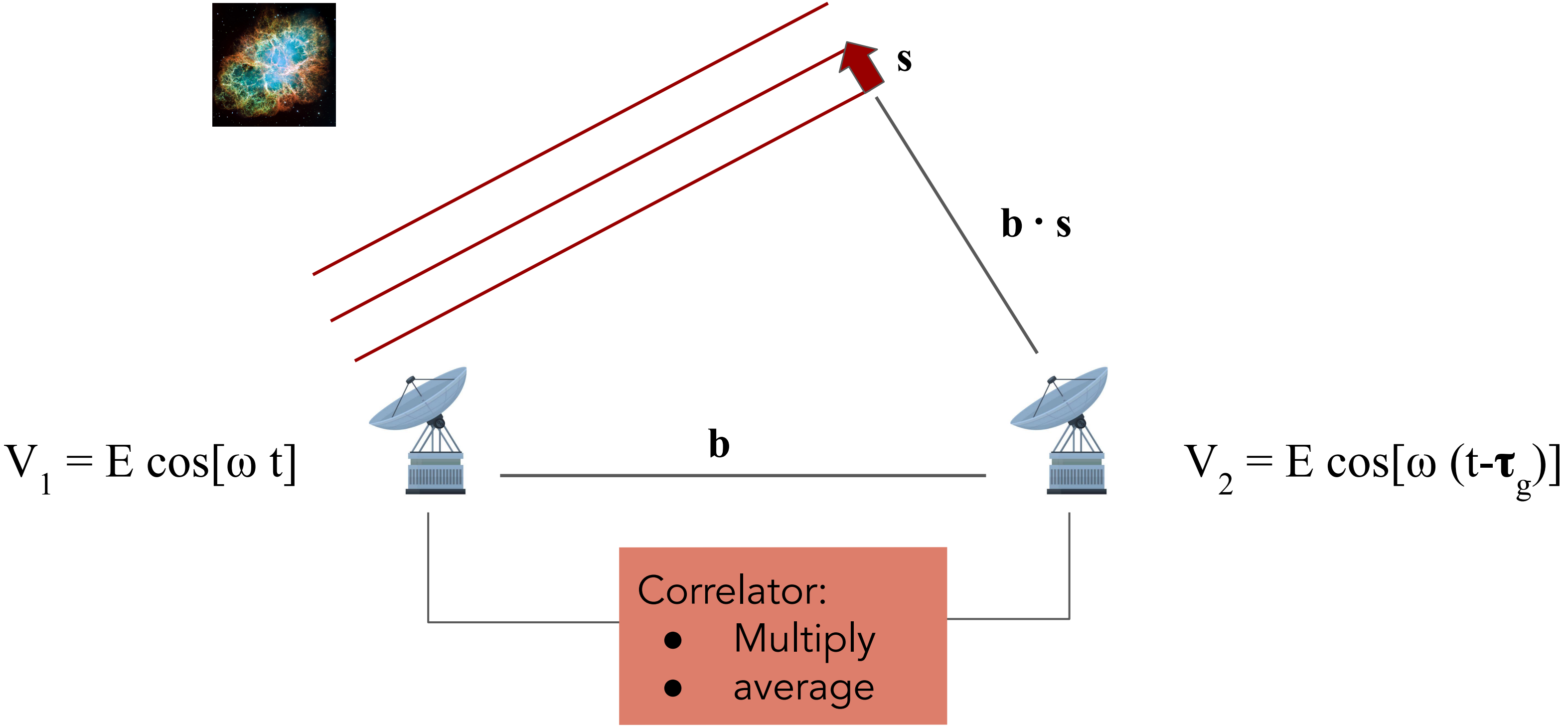
**A lot of the following content will follow from the NRAO Synthesis Imaging Workshop slides*

The most basic interferometer

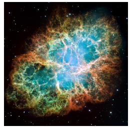
Geometric delay:
$\boldsymbol{\tau}_g = \mathbf{b} \cdot \mathbf{s} / c$

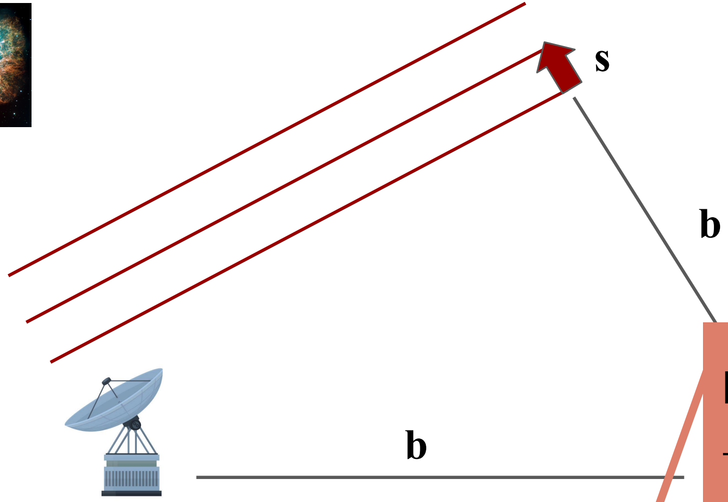


The most basic interferometer



The most basic interferometer





$V_1 = E \cos[\omega\, t]$

Correlator:
- Multiply
- average

Multiply:

$\rightarrow E \cos[\omega\, t] * E \cos[\omega\, (t-\boldsymbol{\tau}_g)]$

$\rightarrow \frac{1}{2} E^2 (\cos[\omega\, \boldsymbol{\tau}_g] + \cos[2\omega t - \omega\boldsymbol{\tau}_g)]$

$\rightarrow P(\cos[\omega\, \boldsymbol{\tau}_g] + \cos[2\omega t - \omega\boldsymbol{\tau}_g])$

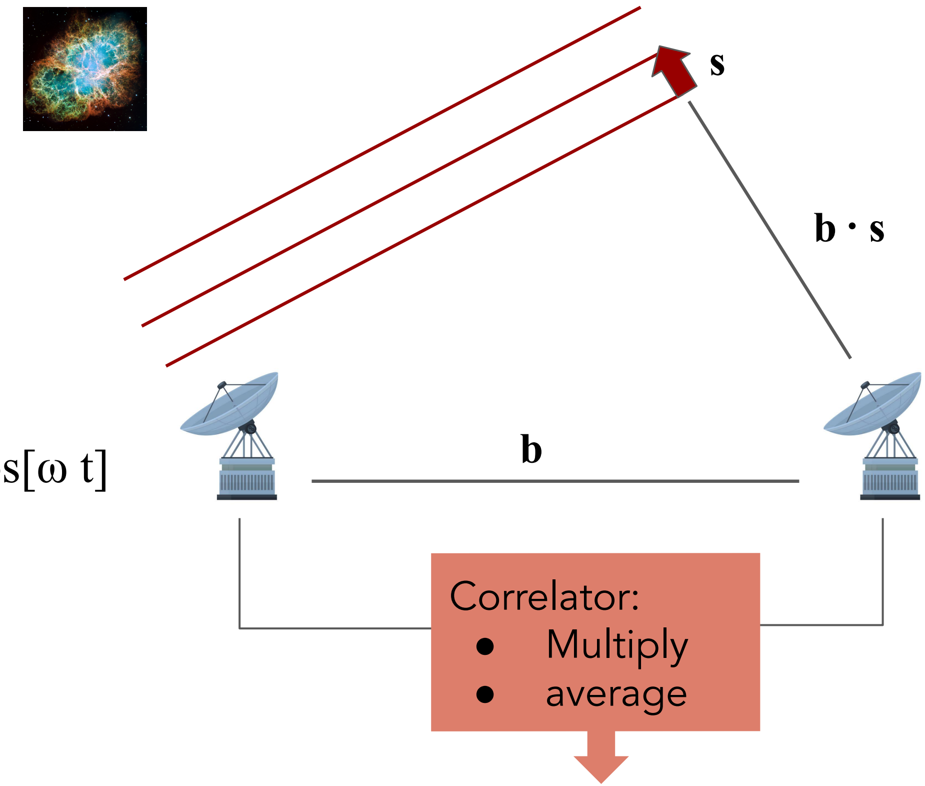
Average:

$\cos[2\omega t - \omega\boldsymbol{\tau}_g]$ is rapidly varying

$\rightarrow P(\cos[\omega\, \boldsymbol{\tau}_g]\,)$

The most basic interferometer

$V_1 = E \cos[\omega\, t]$



$V_2 = E \cos[\omega\, (t - \boldsymbol{\tau}_g)]$

$$
\begin{aligned}
R_c &= P \cos[\omega\, \boldsymbol{\tau}_g] \\
&= P \cos[\omega\, \mathbf{b} \cdot \mathbf{s} / c] \\
&= P \cos[2\pi\, \mathbf{b} \cdot \mathbf{s} / \lambda]
\end{aligned}
$$

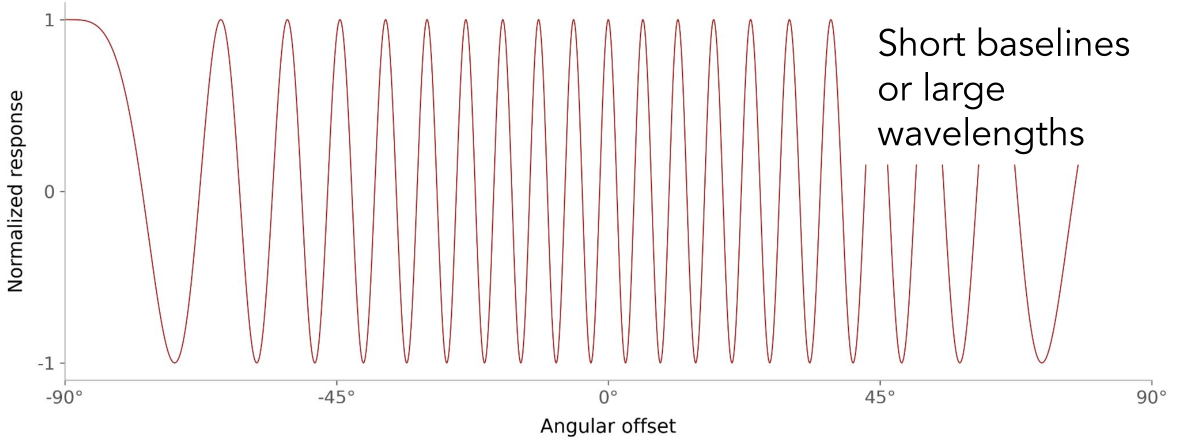
Fringe pattern

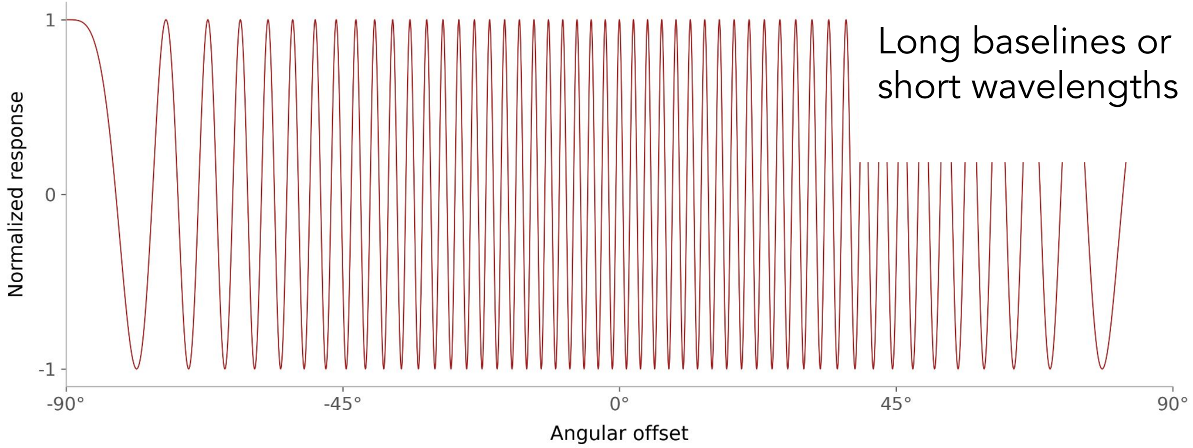
$$R_c = P \cos[2\pi \, \mathbf{b} \cdot \mathbf{s} / \lambda]$$

Response

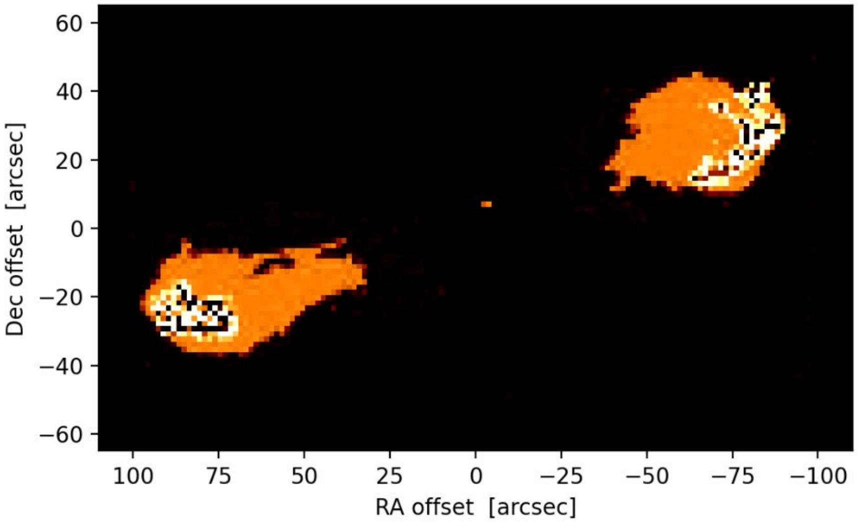
Electric field

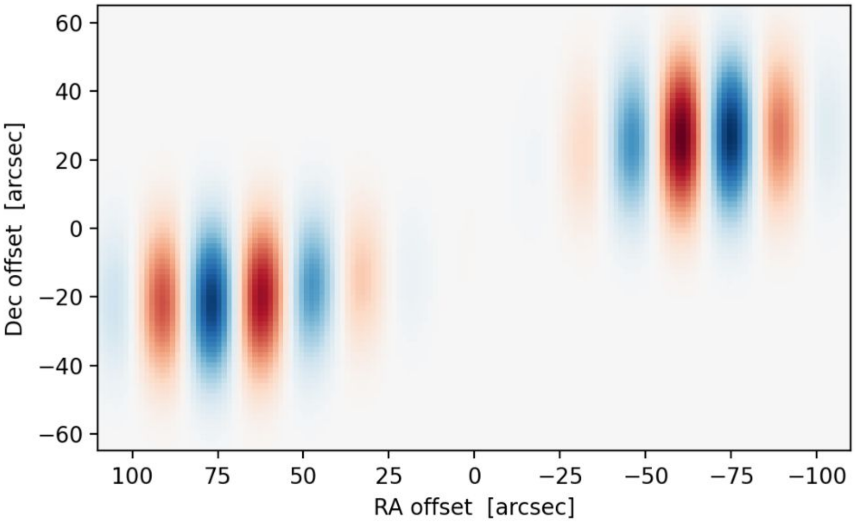
Geometric delay





Fringe pattern, applied to Cygnus A





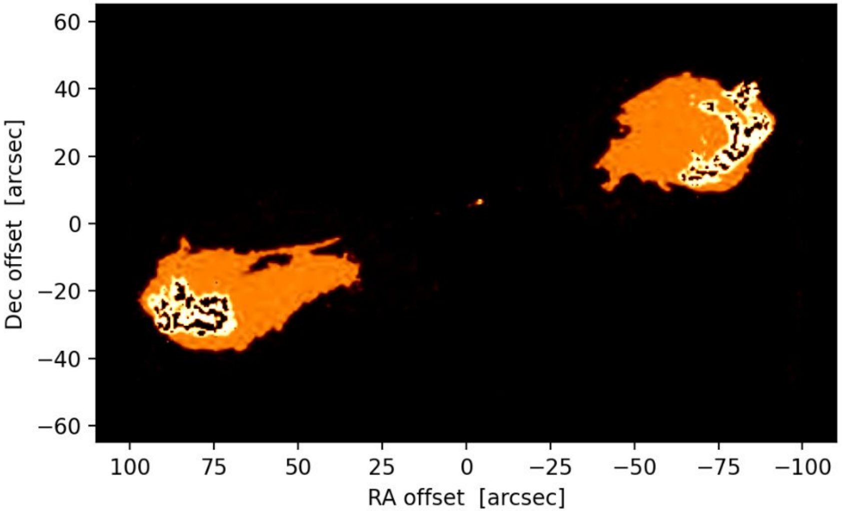
Theoretical interferometer:

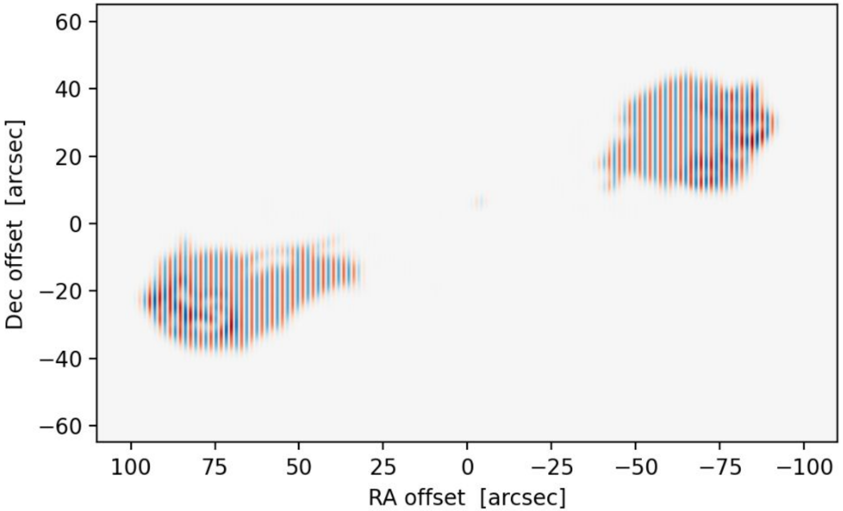
- 400 MHz
- 5km E-W baseline, so 30″ resolution

→ 5% of total flux recovered

Simulations made with the help of Claude

Fringe pattern, applied to Cygnus A





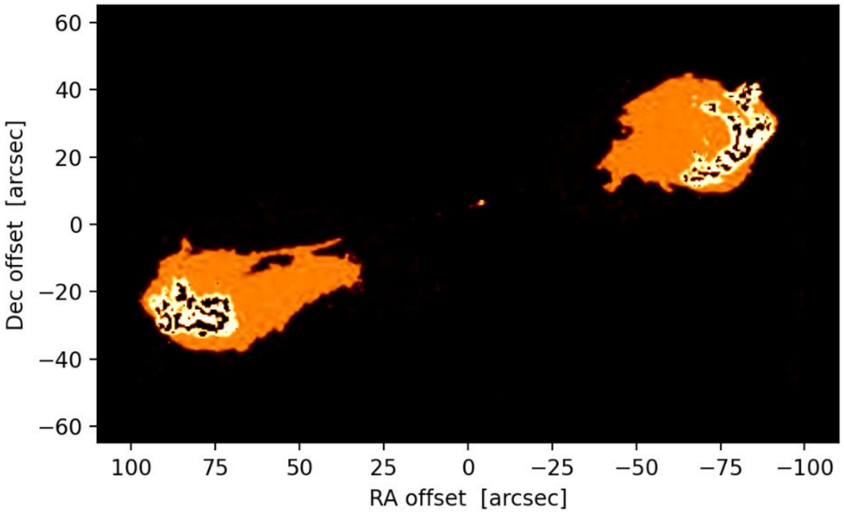
Theoretical interferometer:

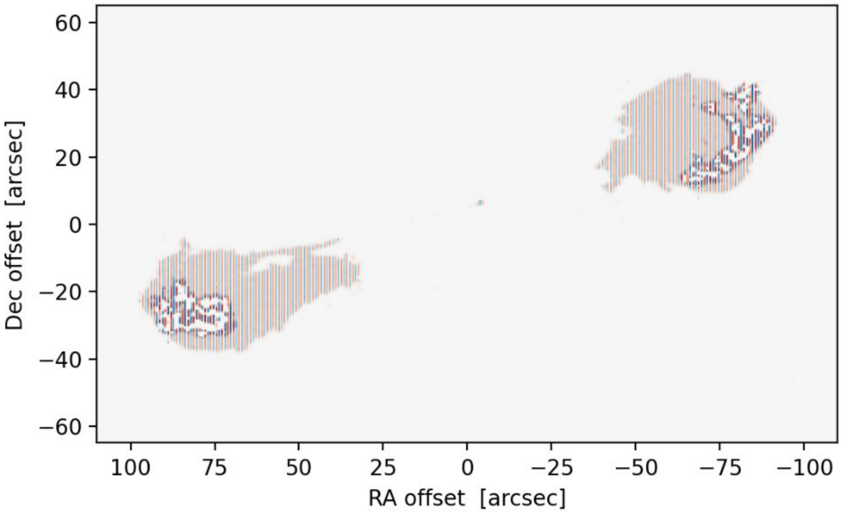
- 400 MHz
- 50km E-W baseline

→ 1% of total flux recovered

Simulations made with the help of Claude

Fringe pattern, applied to Cygnus A





Theoretical interferometer:

- 400 MHz
- 1000km E-W baseline

→ 0.1% of flux recovered, and the source is nearly *resolved out*

Simulations made with the help of Claude

So far, we’ve got a simple cosine interferometer. Why might this not be so good?

Ask students to reflect in groups on this for 5 minutes

So far, we've got a simple cosine interferometer. Why might this not be so good?

$R_c \propto \cos[2\pi\, \mathbf{b} \cdot \mathbf{s} \,/\, \lambda]$

If our source is extended, we integrate this over the source's intensity and sky angle dΩ such that:

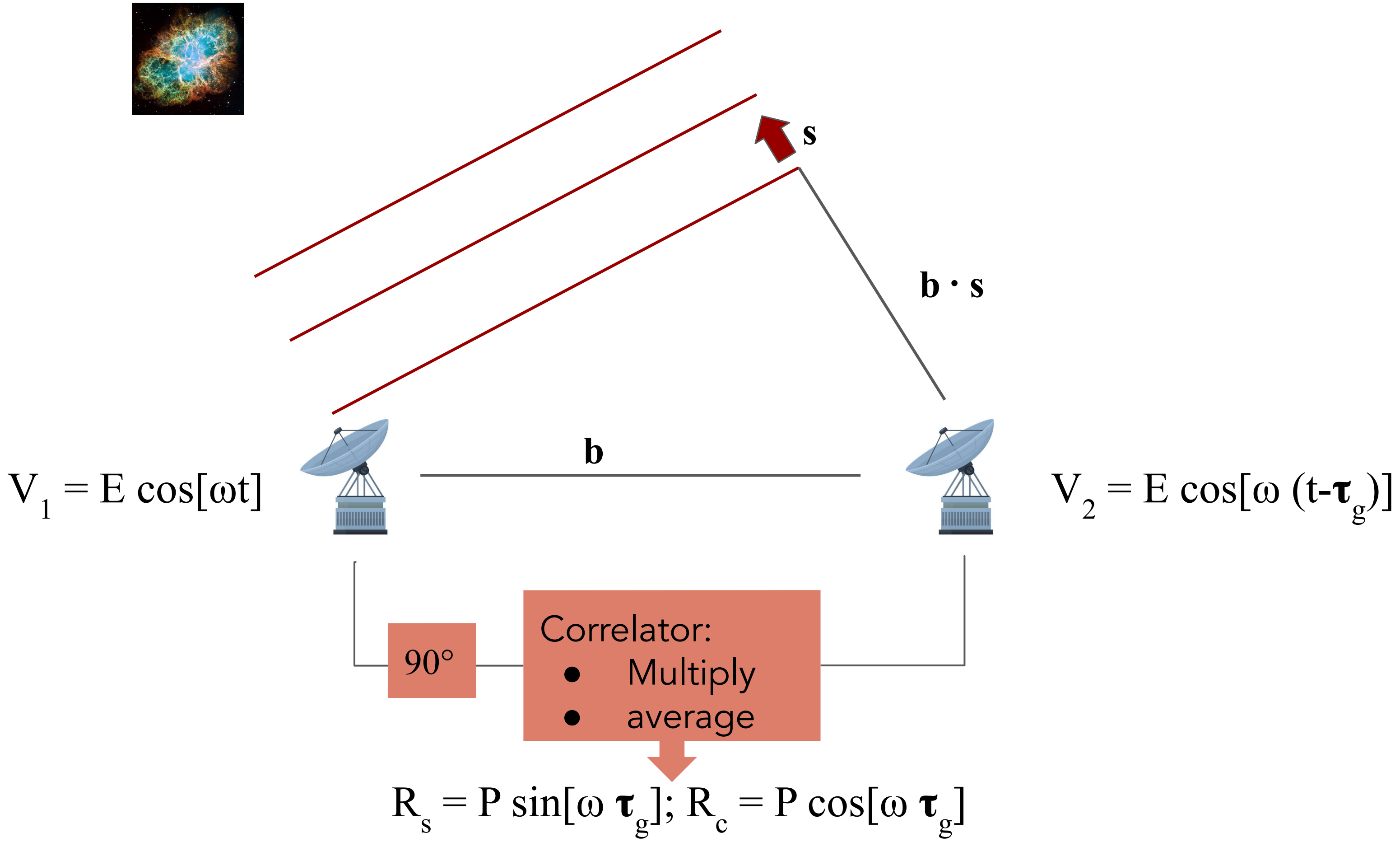
$R_c = \iint I(s) \cos(2\pi\nu\, \mathbf{b} \cdot \mathbf{s}/c)\, d\Omega$

Cosine function is *even*, meaning this is only sensitive to the even part of $I(s)$ e.g.,

$R_c = \iint I_0(s) \cos(2\pi\nu\, \mathbf{b} \cdot \mathbf{s}/c)\, d\Omega = 0$

But, real sources on the sky have both even and odd components. Need to create a new system that can catch both the even and odd components.

The complex correlator



Complex correlator → the complex visibility

$$V(\mathbf{b}) = R_c - iR_s$$

$$= P\cos[\omega\, \boldsymbol{\tau}_g] - i{*}P\sin[\omega\, \boldsymbol{\tau}_g]$$

$$= \iint I(s)\, e^{-2\pi i \nu \mathbf{b} \cdot \mathbf{s}/c} d\Omega$$

Complex correlator → the complex visibility

$$V(\mathbf{b}) = R_c - iR_s$$

$$= P\cos[\omega\, \boldsymbol{\tau}_g] - i{*}P\sin[\omega\, \boldsymbol{\tau}_g]$$

$$= \iint I(s)\, e^{-2\pi i\nu \mathbf{b}\cdot\mathbf{s}/c} d\Omega$$

Notice: this is just a fourier transform between the sky brightness I(s) and our measurements V(b). This is often known as the Van Cittert-Zernike Theorem.

$$\boxed{\begin{aligned} V(\mathbf{b}) &= \iint I(\mathbf{s})\, e^{-2\pi i\nu \mathbf{b}\cdot\mathbf{s}/c} d\Omega \\ I(\mathbf{s}) &= \iint V(\mathbf{b})\, e^{2\pi i\nu \mathbf{b}\cdot\mathbf{s}/c} d^2 b \end{aligned}}$$

Required assumptions:

- spatial incoherence
- source is in the far-field

Things have been rather simplified…

So far, we have had a very simple interferometer. What are some of the biggest things that will breakdown here?

No group breakout but ask students what they think will be the biggest worries

Things have been rather simplified…

So far, we have had a very simple interferometer. What are some of the biggest things that will breakdown here?

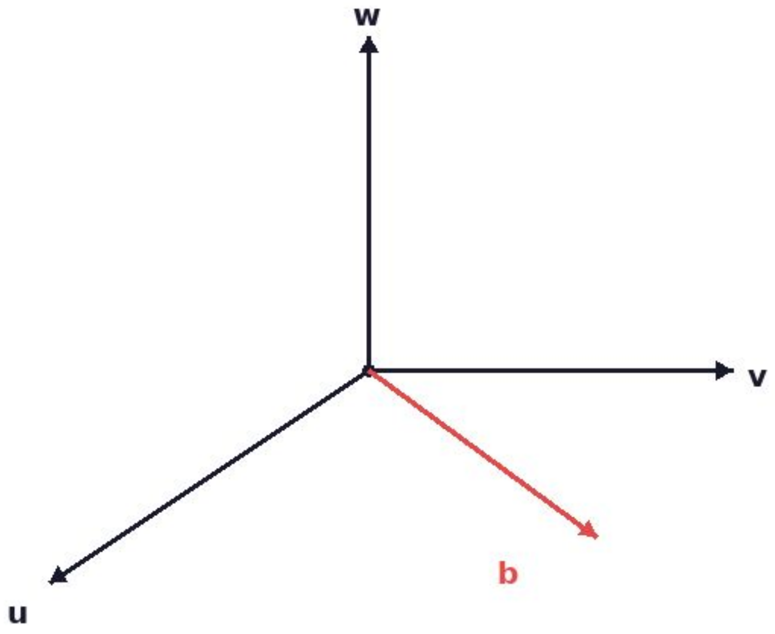
1. 1D meets 2D
2. Finite bandwidth
3. Earth is rotating
4. Antenna pattern

Planar interferometer: introducing the UV-plane

Define a coordinate system: **u**, **v**, **w**

Let baseline **b** lie in the U-V plane, with zero component in the **w**-direction. We then define **b** as:

b = (u, v, 0) where **u, v** are always defined in terms of wavelengths



Planar interferometer: introducing the UV-plane

Next, let's define **s** in this coordinate system. We define **s** using (l, m, n). These are referred to as the direction cosines.

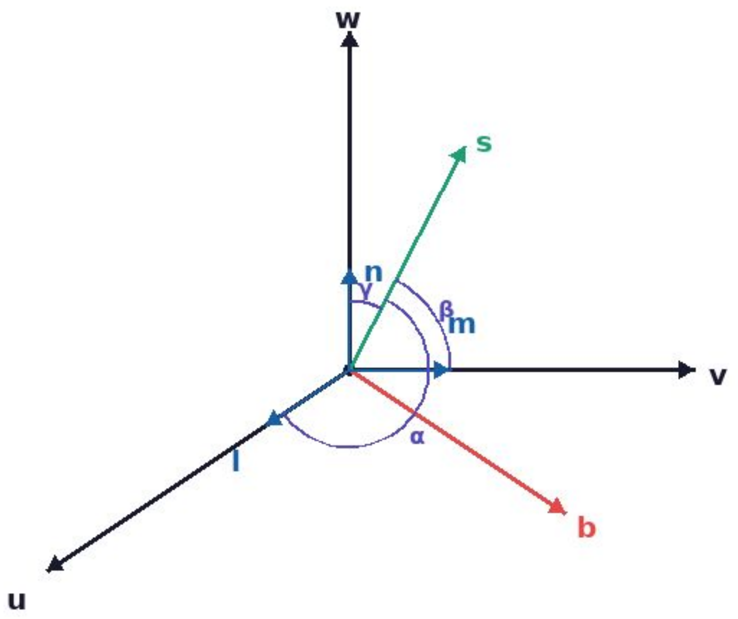
With this:

$\mathbf{b} \cdot \mathbf{s} / \lambda = ul + uv + wn = ul + uv$

(in our simplified case of $w = 0$)

$V(u, v) = \iint I(l, m)\, e^{-2\pi i \nu (ul + vm) / c}\, dl\, dm$

$$\boxed{V(u, v) \Leftrightarrow I(l, m)}$$



Planar interferometer: a 2D interferometer

How realistic is this?

- Some arrays are *truly* aligned like this e.g., Westerbork Synthesis Radio Telescope, Australia Telescope Compact Array, DRAO Synthesis Radio Telescope
- Or they are 2D for short periods of time in snapshot modes e.g., VLA, MeerKAT

What if this doesn't hold?

- Can still approximate an array as this given a few key notes:
  - Align w-axis to point towards the source, u towards east and v towards NCP
  - If source close to phase centre, can approximate as a 2D array
  - Clark condition: Need to stay close enough to your pointing centre, otherwise this breaks down

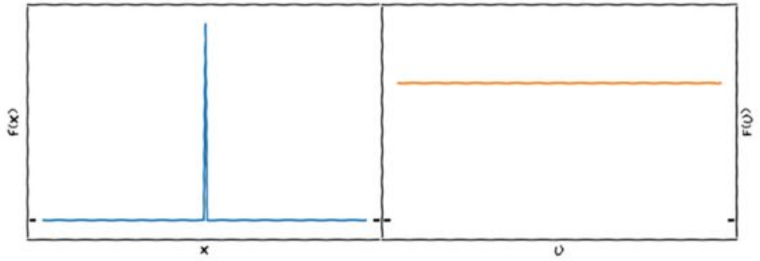
Planar interferometer: sampling the UV-plane

$$V(u, v) \Leftrightarrow I(l, m)$$

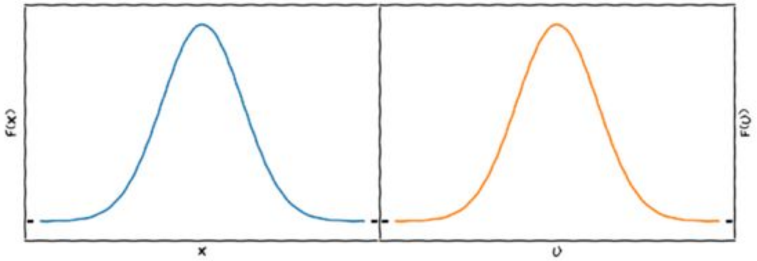
Sampling the visibility at a given (u, v) point corresponds to the FT of sampling the sky brightness at a given point. If we want to sample the full sky, we need to sample the UV-plane.

How can we do this?
→ more baselines, sampling of these baselines, and frequency coverage
→ taking advantage of Earth's rotation and hence the rotation of our baselines with respect to a source's position

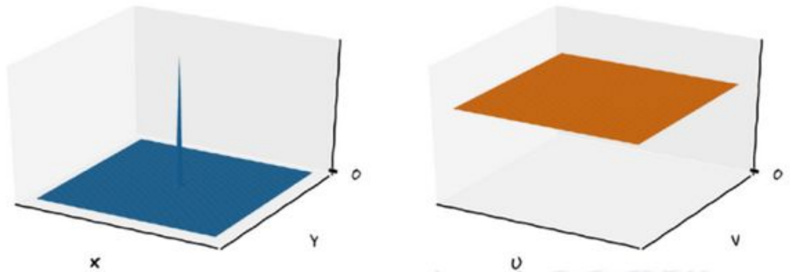
Brief reminder on Fourier Transforms



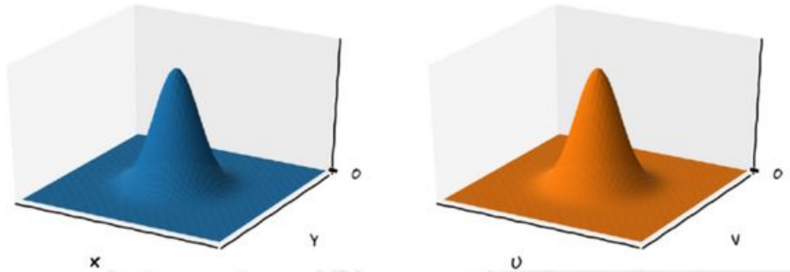
A Delta function is transformed into a constant.
* If offset from x = 0, then into a sinusoidal function.



A Gaussian function is transformed into another Gaussian function.



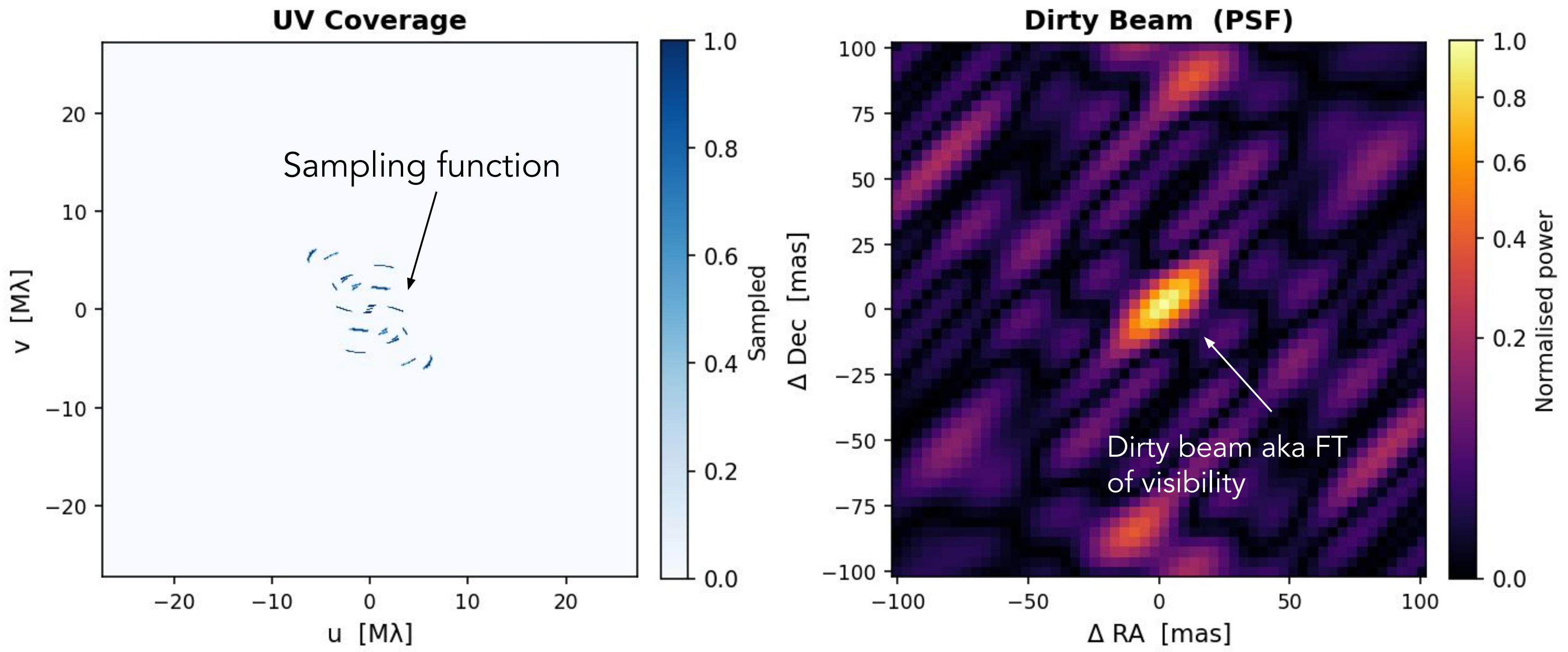
A 2D Delta function is transformed into a plane (any U, V point sees the same value).



A 2D Gaussian function is transformed into a 2D Gaussian.

Slide from Shami Chatterjee & Florian Eppel

Sampling the UV-plane: short baselines, good coverage

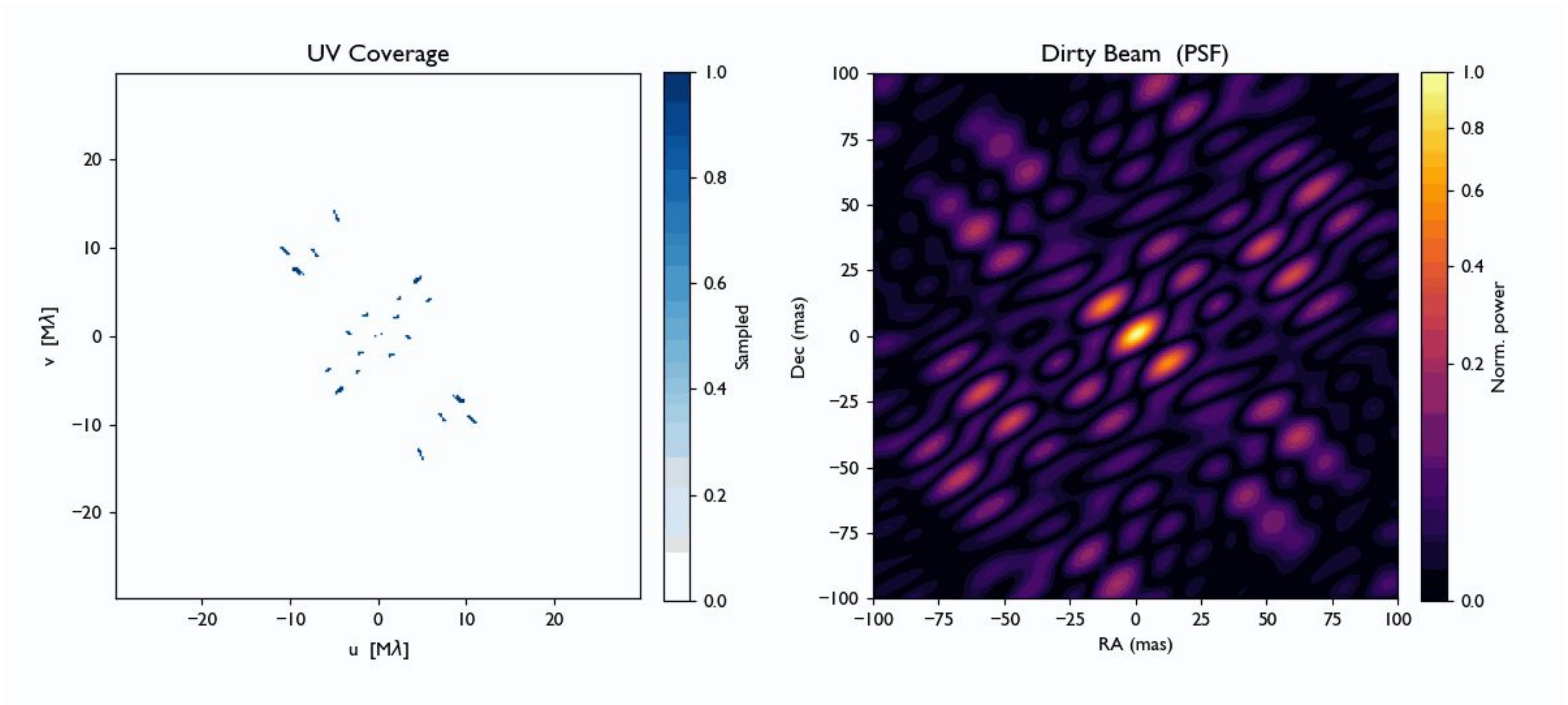


Theoretical interferometer:

- 1 GHz
- Short baselines
- 128 MHz bandwidth
- 1hr integration

Simulations made with the help of Claude

Sampling the UV-plane: long baselines, uneven coverage

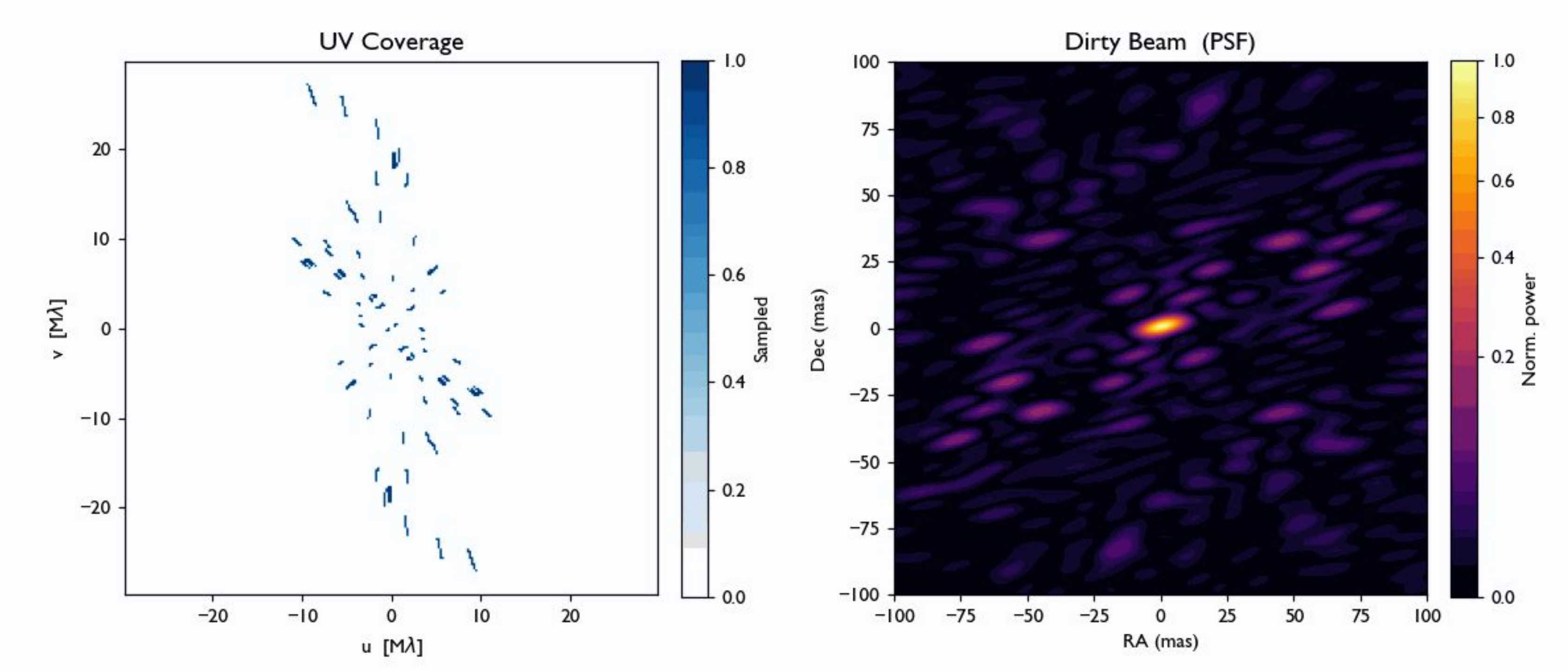


Theoretical interferometer:

- 1 GHz
- 6/10 VLBA antennas
- 128 MHz bandwidth
- 1hr integration

Simulations made with the help of Claude

Sampling the UV-plane: long baselines, good coverage



Theoretical interferometer:

- 1 GHz
- VLBA antennas
- 128 MHz bandwidth
- 1hr integration

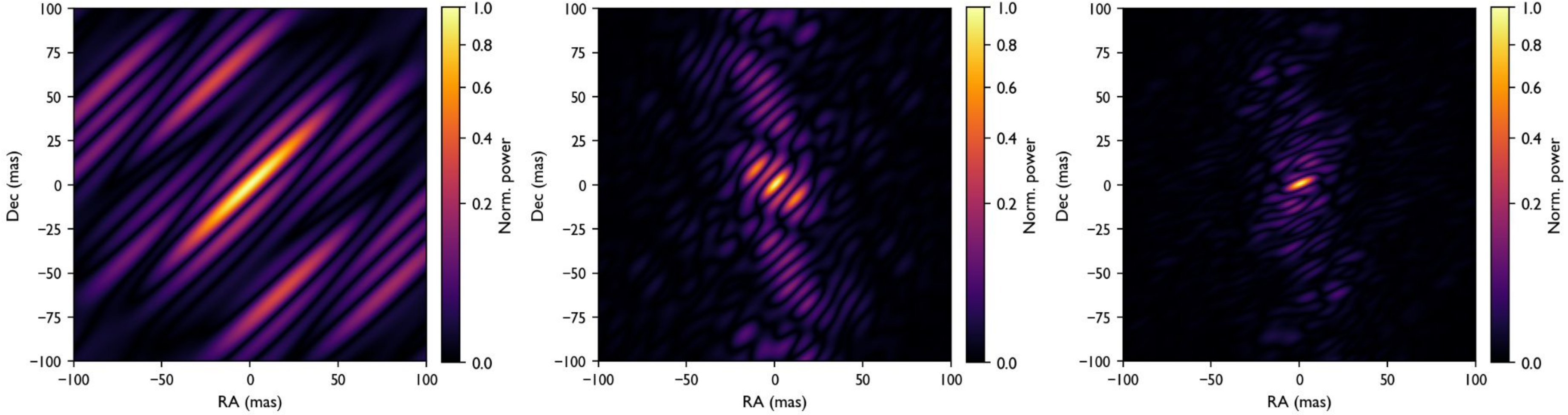
Simulations made with the help of Claude

Sampling the UV-plane: long baselines, good coverage, doubled frequency

Short baselines

Long baselines, missing antennas

Long baselines, full coverage



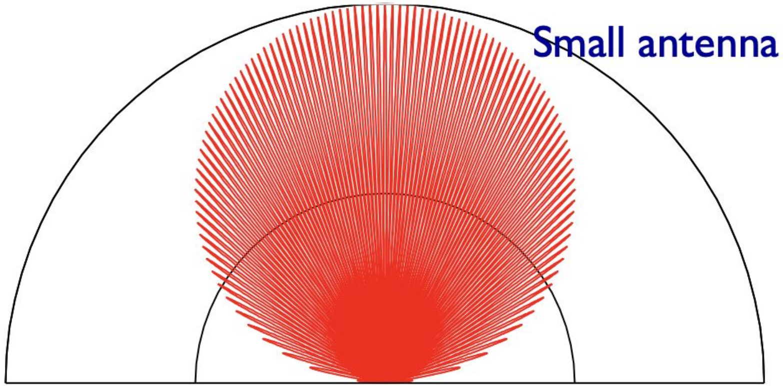
Simulations made with the help of Claude

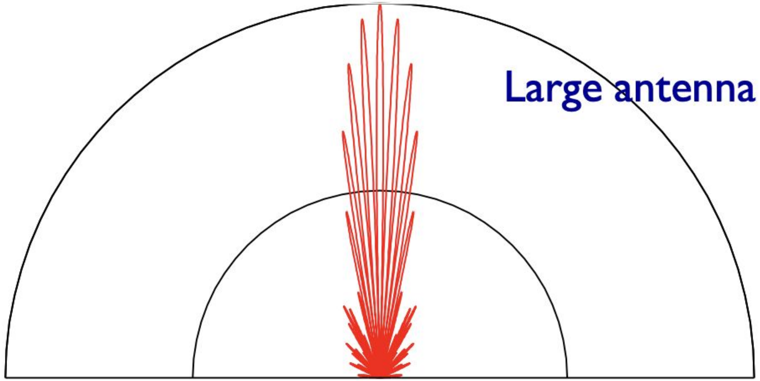
Antenna pattern

Real sensors have a power pattern.
Let's call this A. Then,

$$V(\mathbf{b}) = \iint A_1 A^*_2 \, I(\mathbf{s}) \, e^{-2\pi i \nu \mathbf{b}.\mathbf{s}/c} \, d\Omega$$

Result? Attenuate your fringe pattern by your antenna power





Finite bandwidth

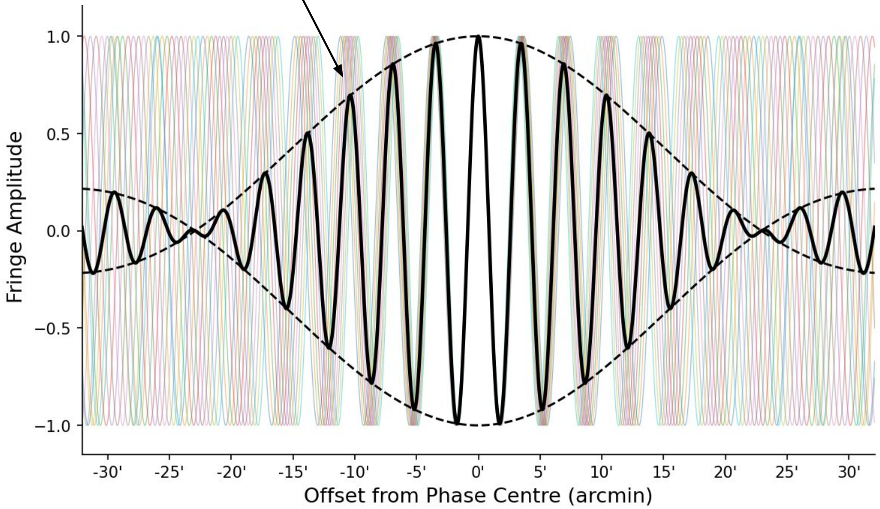
→ Each frequency component will have its own fringe pattern *but* the maximum will be shared amongst different frequencies

→ Integrate our visibility over a small frequency range $\Delta\nu$ to find that the fringe function is attenuated by $\mathrm{sinc}(\tau_g \Delta\nu)$

→ Nulls happen at $\tau_g \Delta\nu = 1$

Instrument	$\Delta\nu$	baseline	Null location
VLA	1MHz	35 km	30arcmin
VLBA	1MHz	3000 km	20mas

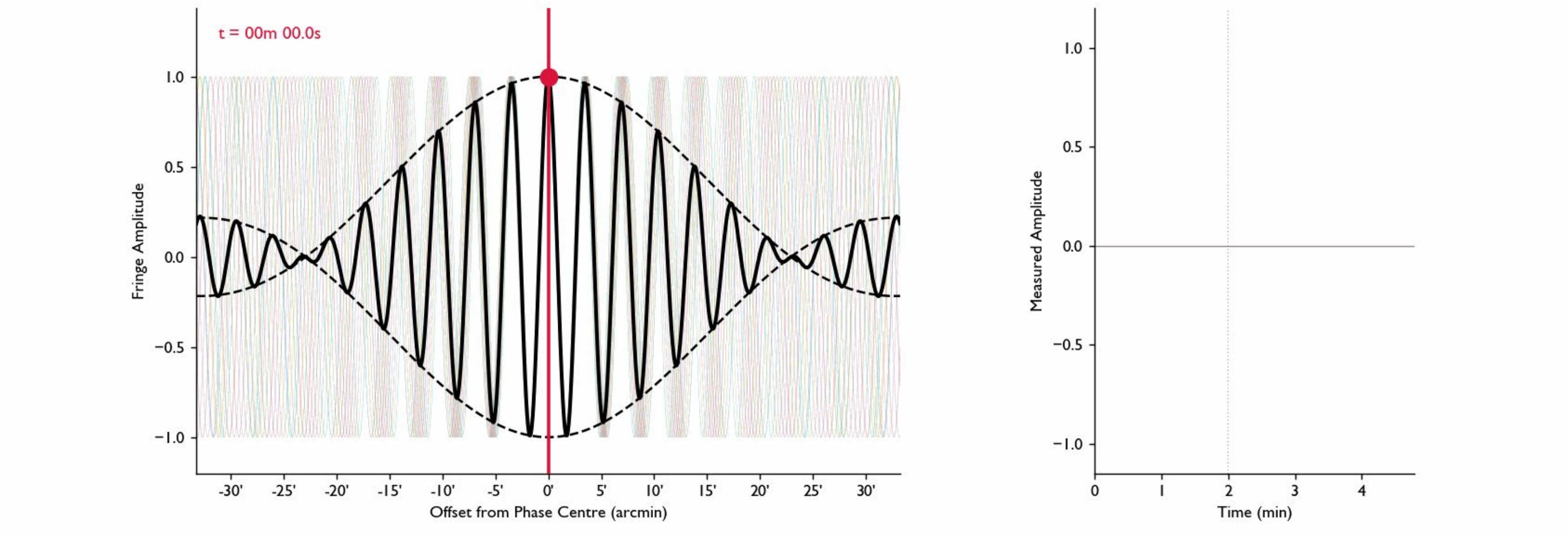
Sinc attenuation function



Rotation of the Earth

Need to account:

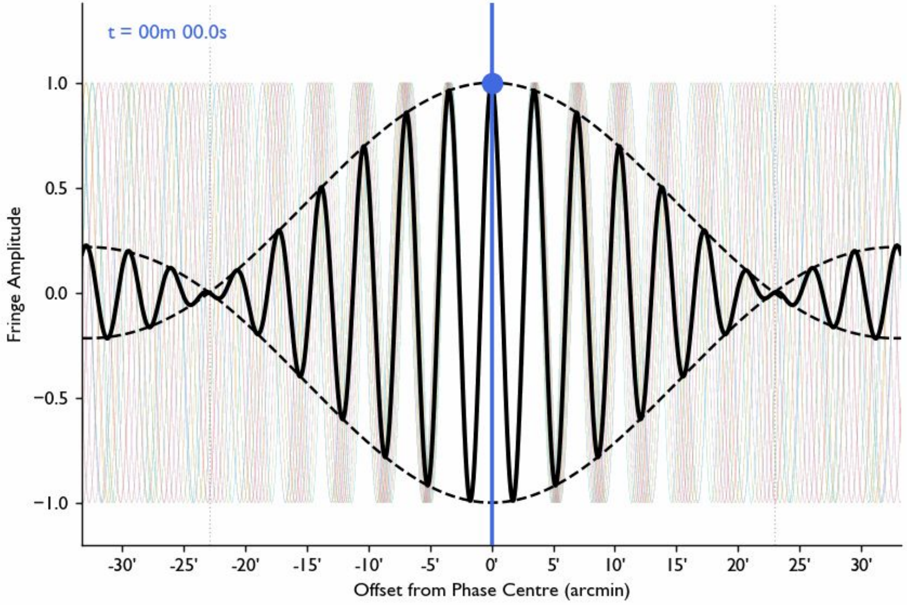
- Long baselines → fast fringes
- As Earth rotates, source will move through the fringes *unless* you correct for this

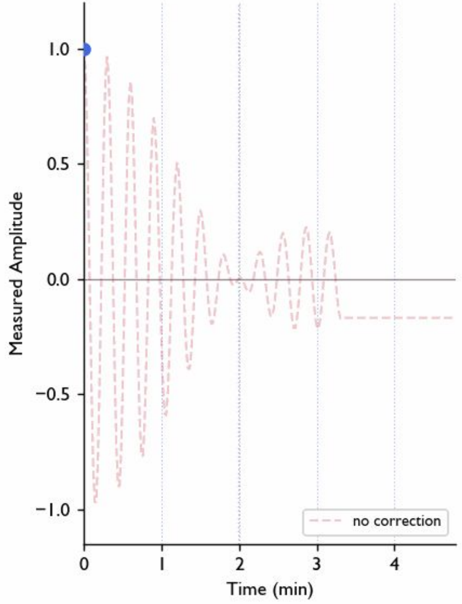


Rotation of the Earth

How do you fix this?

- Add back in an oscillating sine wave with the fringe rate from Earth's rotation at the phase centre





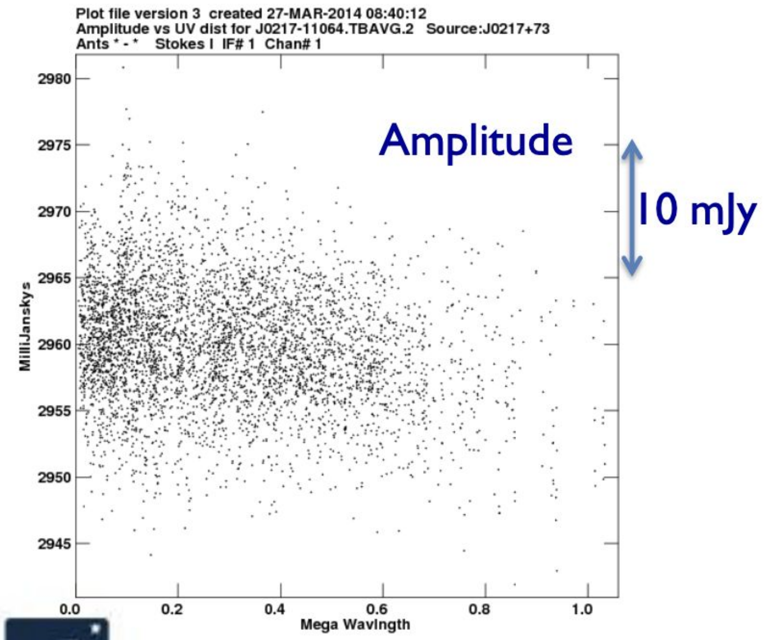
Summary

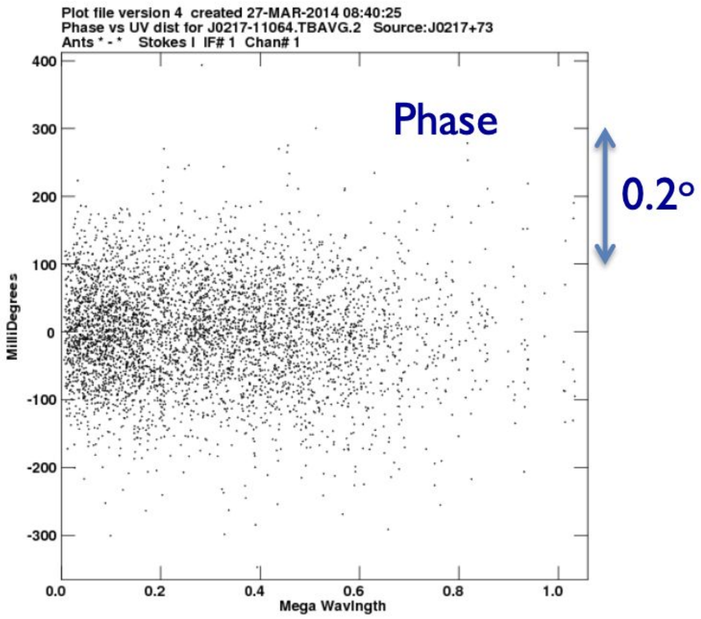
If you have spaced out for the entire lecture, here's what I hope you can remember:

- **Visibilities:** measured, correlated electric fields (with many additional effects from e.g., sampling, bandwidth, etc)
- $V(u, v) \Leftrightarrow I(l, m)$ – The visibility is the FT of your sky brightness
- Your UV-coverage (aka your baseline orientations) determine your dirty beam, which will convolve with the sky brightness to give you your final image
- Long baselines – improved resolution; short baselines – more flux

If there is time..

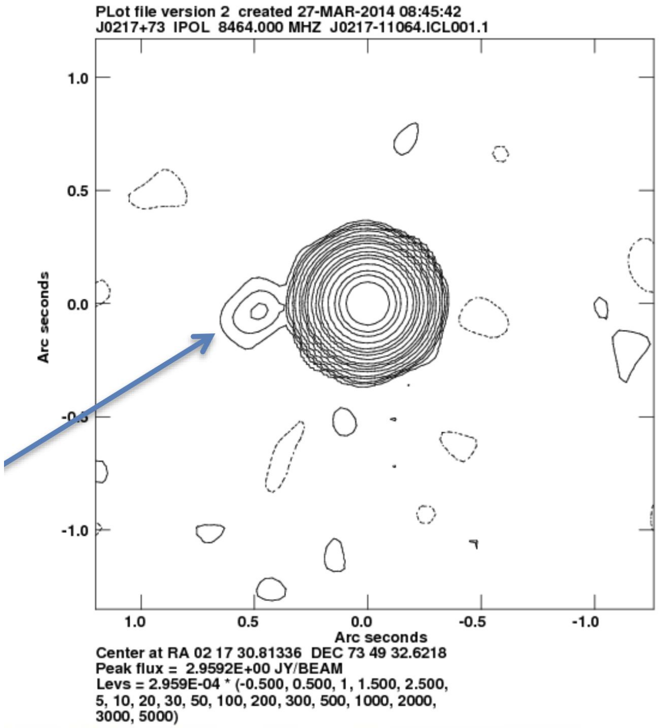
Understanding your source without the image itself





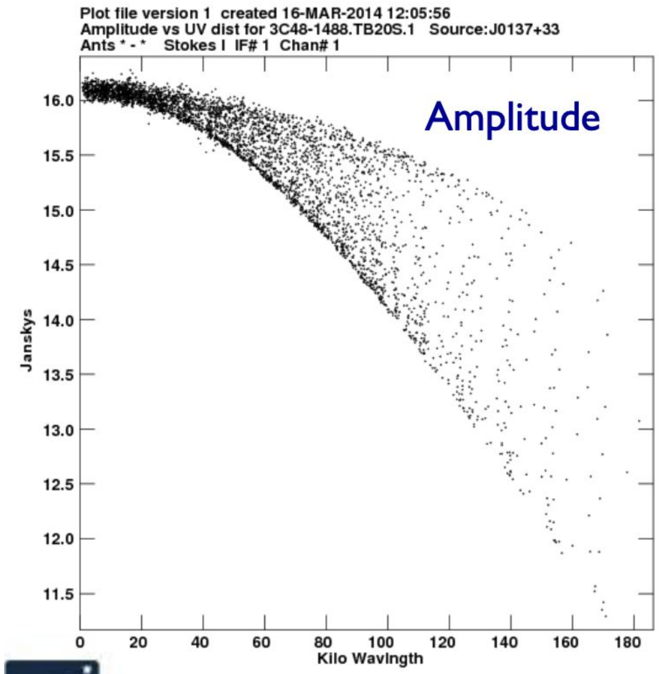
What type of source do we think this is, and why?

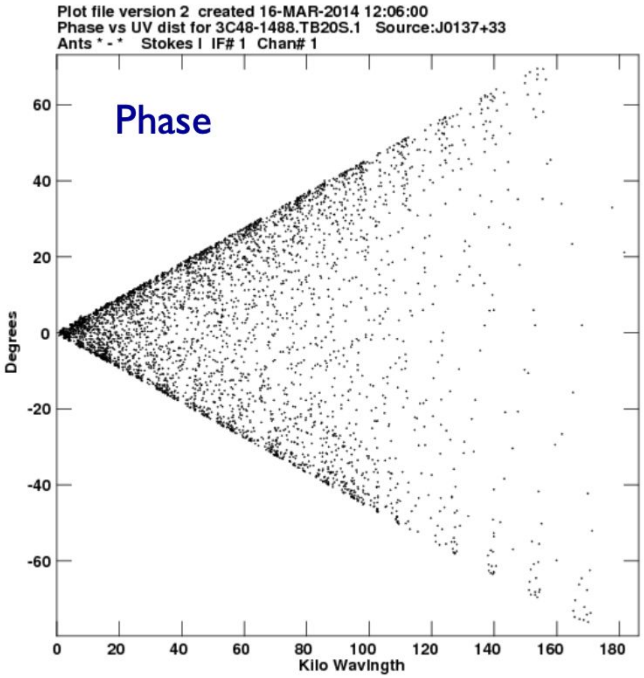
Understanding your source without the image itself



Point source

Understanding your source without the image itself





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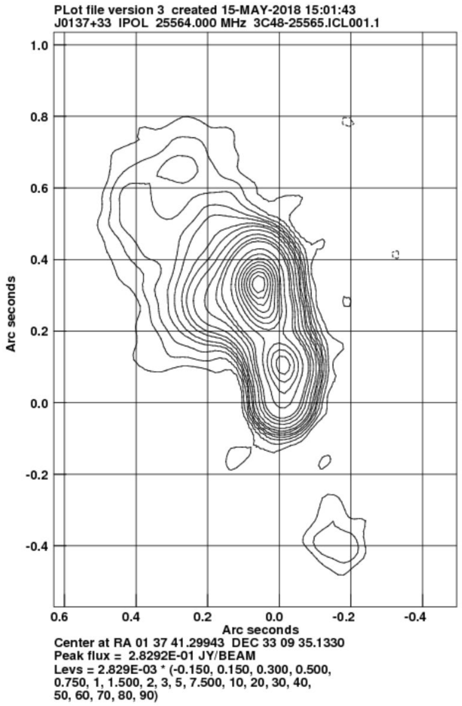
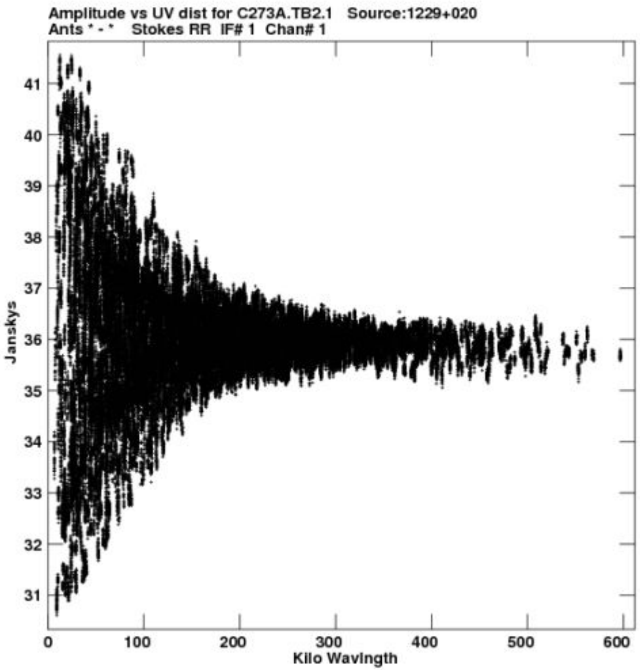
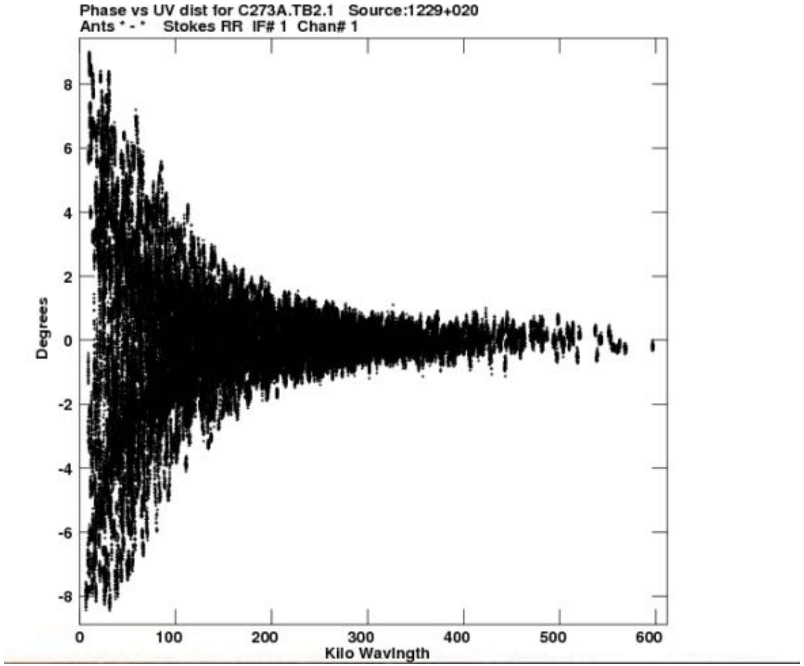
Understanding your source without the image itself



Complex source

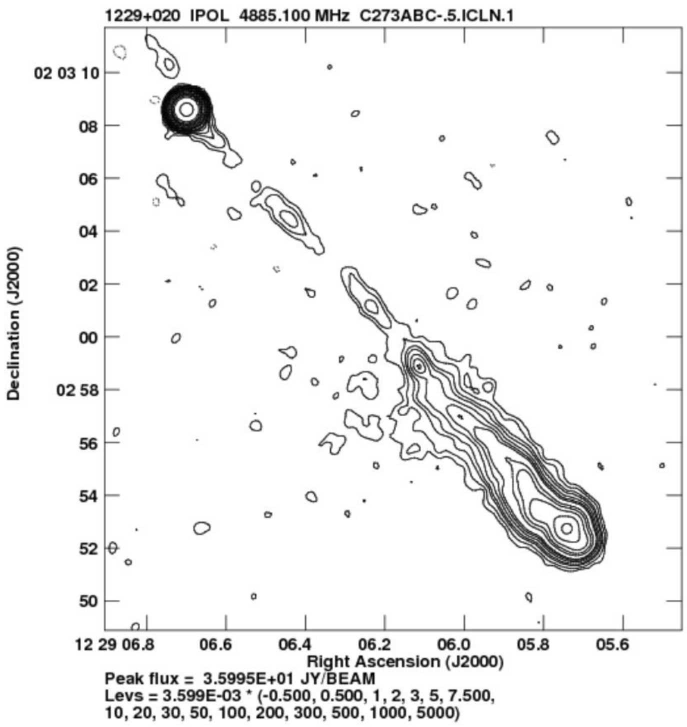
Understanding your source without the image itself





What type of source do we think this is, and why?

Understanding your source without the image itself



AGN