Light tetraquarks mass estimates using QCD sum-rules

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Outline

1. Quantum Chromodynamics
2. Exotic Hadrons
3. Tetraquarks
4. Results
5. Summary

Quantum Chromodynamics

QCD: Quantum Chromodynamics

Part of The Standard Model, theory of fundamental interactions.

- → WHAT IS IT?
  The theory of the strong interactions: quarks and gluons.
- → WHY WAS IT PROPOSED?
  The need to explain the existence of hadrons, such as baryons and mesons.
- → MAIN FEATURE
  Extra degree of freedom denoted by: colour, whose algebra is ruled by the colour gauge symmetry group $SU(3)_C$.

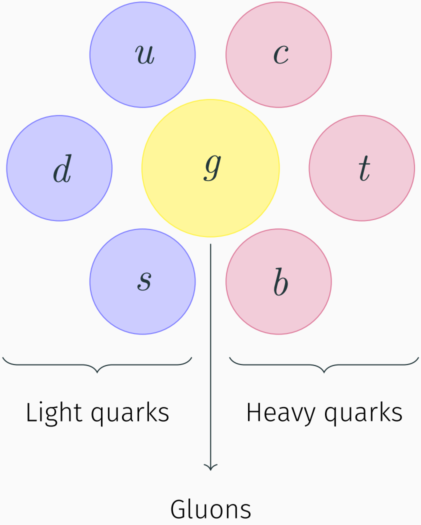
Mathematical Structure: The QCD Lagrangian will be

$$\mathcal{L}_{\rm QCD} = \sum_{j=1}^{6} \overline{\psi}_j^a \left(i\gamma^\mu \left(\partial_\mu \, \delta^{ab} + i g_s \, t_A^{ab} \mathcal{A}_\mu^A\right) - m_j \, \delta^{ab} \right) \psi_j^b - \frac{1}{4} G_{\mu\nu}^A G_A^{\mu\nu},$$

ψ_j^a ($\overline{\psi_j^a}$): quark (antiquark) of flavour j,
$\mathcal{A}_\mu^A$: gluon fields,
$t_{ab}^A = \frac{1}{2}\lambda_{ab}^A$: matrix representation of the generators of the group,
$G_{\mu\nu}^A$: field strength tensors

$$G_{\mu\nu}^A = \partial_\mu \mathcal{A}_\nu^A - \partial_\nu \mathcal{A}_\mu^A + g_s f^{ABC} \mathcal{A}_\mu^B \mathcal{A}_\nu^C.$$

QCD: Particle content



Different types:

→ Common:

1. Baryons: qqq or $\overline{qqq}$.
2. Mesons: $q\bar{q}$.

→ Exotic:

1. Multiquark states: tetraquarks, pentaquarks, etc.
2. Glueballs: gluons.
3. Hybrids: quarks + gluons.

Exotic Hadrons

Multiquark states

Various types of exotic hadrons: **tetraquarks**, pentaquarks, glueballs, hybrids, etc. which satisfy **COLOR NEUTRALITY**.

There is experimental evidence of the existence of multiquark states, such as the recently discovered candidate $X(6900)$[1], $X_0(2866)$[2] and $X_1(2904)$[3] at LHCb.

Objective

Examine higher-loop NLO effects on the mass predictions of multiquark states. Particularly, the effects of these corrections on the tetraquark scalar meson $\sigma(600)$ mass predictions.

[1] R. Aaij, et al. 2006.16957 (2020).
[2] Liu Ming-Zhu, et al. 2008.07389 (2020)
[3] Huag, Yin and Lu, et al. 2008.07959 (2020)

Tetraquarks

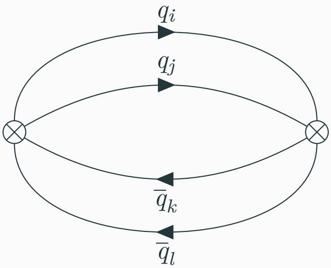


Figure 1: Leading-order Feynman diagram for diquark-antidiquark configuration (Tetraquarks).

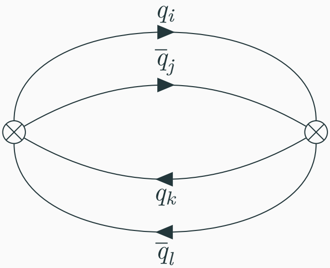
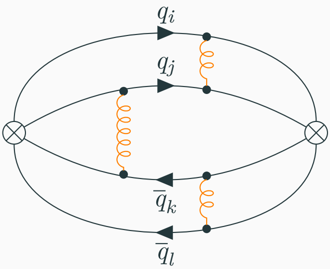
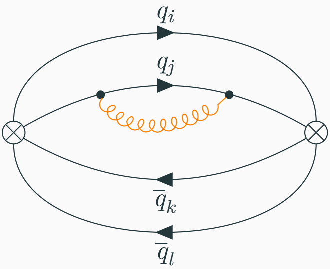


Figure 2: Leading-order Feynman diagram for quark-antiquark configuration (meson-meson).

The tetraquark studied is the scalar meson $\sigma(600)$, whose composition is made of u (up) and d (down) quarks.



(a) Gluon exchange.



(b) Gluon self-interaction.

Figure 3: Next-to-leading order Feynman diagrams for tetraquarks.

From these diagrams one can obtain the Feynman Rules and later, the correlator function, which is given by:

$$\Pi(x) = \langle 0|\mathcal{T}\{J(x)J^\dagger(0)\}|0\rangle.$$

Källen and Lehmann related the two-point functions with the *spectral density* through the dispersion relations as

$$\Pi(Q^2) = \frac{1}{\pi}\int_0^\infty ds \frac{\rho(s)}{s+Q^2}.$$

Correlator

Dispersion relations with the subtraction terms:

$$\Pi(Q^2) = \Pi(0) + Q^2\Pi'(0) + \frac{1}{2}Q^4\Pi''(0) - Q^6\frac{1}{\pi}\int_0^\infty \frac{\rho(s)\,ds}{s^3(s+Q^2)}.$$

Borel transform operator is related to the inverse Laplace transform

$$\hat{B} \equiv \lim_{\mathrm{N,Q^2}\to\infty} \frac{(-Q^2)^N}{\Gamma(N)}\left(\frac{d}{dQ^2}\right)^N.$$

Hence, after applying the Borel transform operator to the correlator, the Laplace Sum rule:

$$R_k(\tau, s_0) = \frac{1}{\pi}\int_0^{s_0} ds\; s^k e^{-s\tau}\rho^{had}(s).$$

Finally, the mass estimate

$$\frac{1}{R_0(s,\tau)}\frac{-dR_0(s,\tau)}{d\tau} = \frac{R_1(s,\tau)}{R_0(s,\tau)} = M_X^2.$$

Results

The spectral function can be separated as follows:

$$\rho(s) = \rho_{had}\delta(s - s_0) + \rho_{cont}(s_0 < s < \infty)$$

Hence, considering a benchmark where the low energy contribution is larger than the continuum. This function[4] will be:

$$\begin{aligned}\rho^{\sigma}(s) =& A_0 s^4 + A_1 s^3 + A_2 \langle g^2 GG \rangle s^2 + A_3 \langle \bar{q}q \rangle s^2 \\ &+ A_4 \langle g^2 GG \rangle s + A_5 \langle g^2 GG \rangle s + A_6 \langle \bar{q}q \rangle s \\ &+ A_7 \langle \bar{q}q \rangle^2 + A_8 \langle g^2 GG \rangle \langle \bar{q}q \rangle + A_9 \langle \bar{q}\sigma Gq \rangle.\end{aligned}$$

[4]H. Chen & Hosaka et al. ”Light scalar tetraquark mesons in the QCD sum rule.” arXiv: 0707.4586 (2007)

Spectral function by its contributions

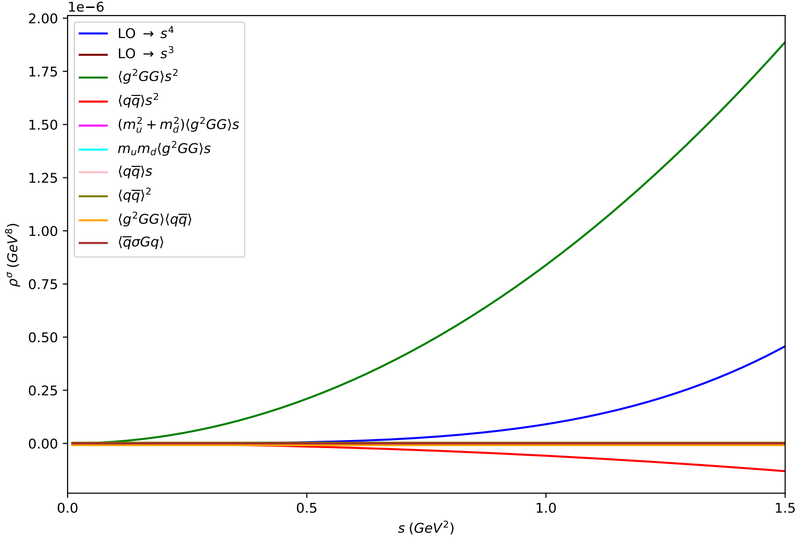


Figure 4: Spectral function versus energy scale per each contribution.

Which can be reduced to,

$$\rho^{\sigma}(s) = A_0 s^4 + A_2 \langle g^2 GG \rangle s^2 + A_3 \langle \overline{q} q \rangle s^2 + A_8 \langle g^2 GG \rangle \langle \overline{q} q \rangle.$$

Now, if we add the NLO terms[5] to the spectral functions, we have:

$$\rho^{\sigma}(s, \mu) = \rho^{\sigma}_{LO}(s) + \rho^{\sigma}_{NLO}(s, \mu),$$

$$\rho^{\sigma}(s) = \frac{1}{11520\pi^6} s^4 \left\{ 1 + \alpha_s \left[\frac{1381 + 15\sqrt{2}}{180\pi} - \frac{7 + 6\sqrt{2}}{4\pi} \log\left(\frac{s}{\mu^2_{\overline{MS}}} \right) \right] \right\}$$
$$+ \left(\frac{6\sqrt{2} + 7}{9216\pi^6} \langle g^2 GG \rangle + \frac{m_u + m_d}{36\pi^4} \langle \overline{q} q \rangle \right) s^2$$
$$+ \frac{(6\sqrt{2} + 1)(m_u + m_d)}{1152\pi^4} \langle g^2 GG \rangle \langle \overline{q} q \rangle.$$

[5]S. Groote, et al. "Perturbative $O(\alpha_s)$ corrections to the correlation functions of light tetraquark currents." arXiv: 1404.4801v2 (2014)

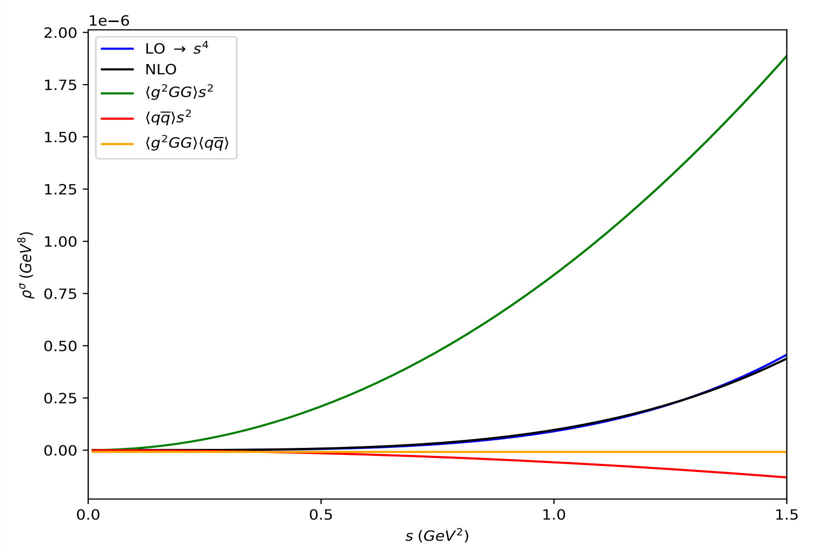


Figure 5: Spectral function versus energy scale per each contribution.

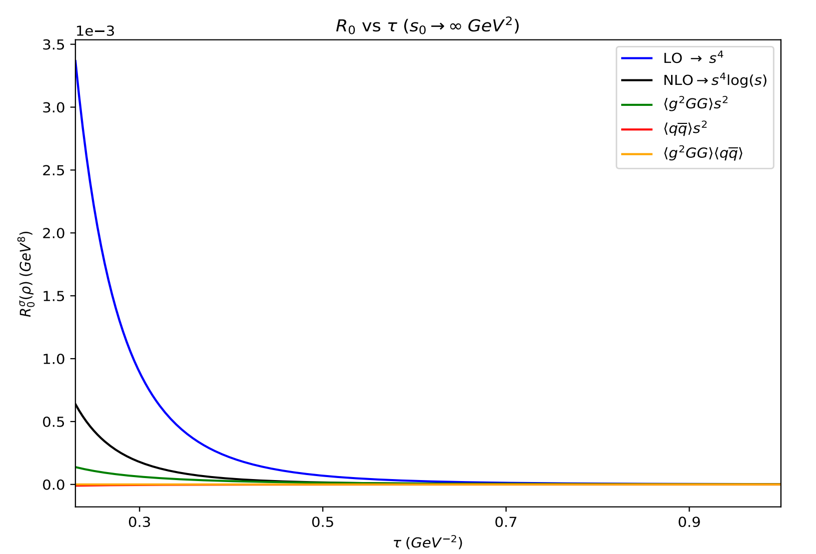


Figure 6: R_0 vs parameter τ when $s_0 \to \infty$.

Sum Rules: R_1

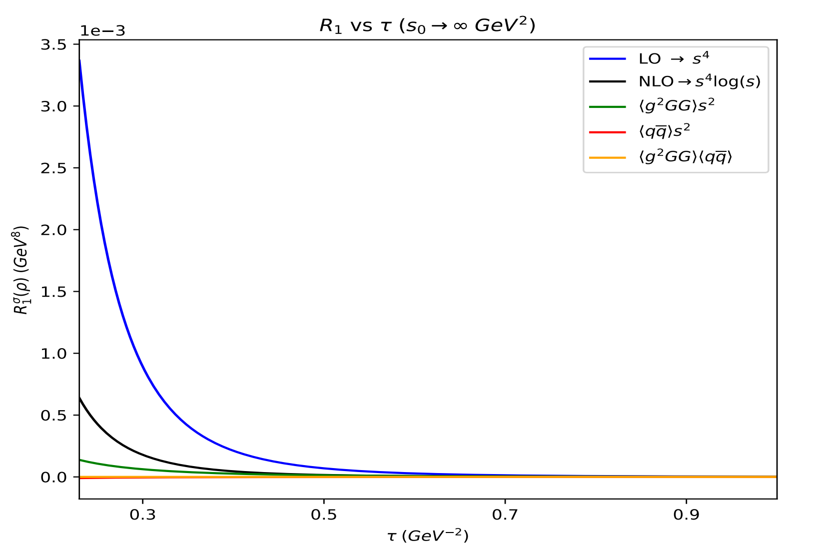


Figure 7: R_1 vs parameter τ when $s_0 \to \infty$.

Conclusions

- The NLO contributions are significant for the scalar tetraquark state σ.
- A whole QCD sum rule re-analysis is required to study the effects of the NLO on the mass predictions.
- This method can be extended for the other light tetraquark states, such as $a_0(980)$, $f_0(980)$, and $\kappa(800)$.

Summary

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1. There is evidence that light tetraquark systems could have large contributions from NLO diagrams. (e.g. Groote, et al.)
2. QCD sum-rules are useful for light tetraquarks systems and the calculation of observables.
3. The contribution from NLO diagrams to light tetraquark systems cannot by neglected.
4. Future work is to obtain mass estimation of these systems, doing a re-analysis of light systems.
5. Develop a method to compute correlation functions of heavier systems.

Thank you!