Orbital Effects in Perturbations around Exotic Compact Objects

Luís Felipe Longo Micchi[1]

[1]Universidade Federal do ABC / Perimeter Institute

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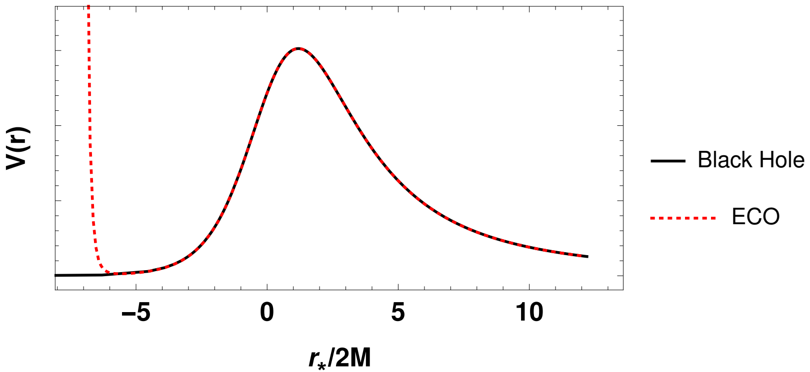
Introduction

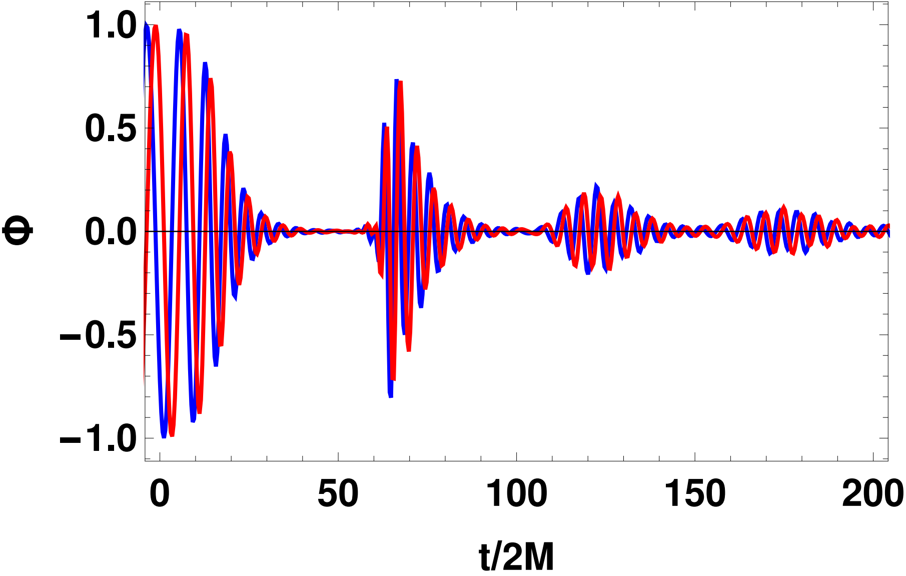
- Black Holes or Exotic Compact Objects (ECOs)?
- ECOs well-approximated by Kerr metric outside horizon
- Partial reflection near (would-be) horizon
- Study the perturbative response for different orbital motions

Introduction



(a) Effective Potential

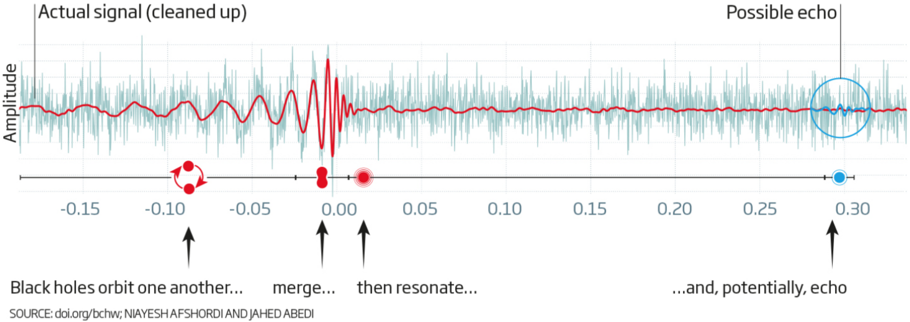
What to expect?



Data: Tentative Evidence?

- Why should we care?
  - Test for quantum nature of black holes
  - Black Hole Information (a.k.a. Firewall) paradox
  - Probe the existence of event horizons
- Do observations support such claims?
  - First tentative evidence: Abedi, Dykaar & Afshordi 2017
  - Several groups found evidence at varying degrees of confidence:
  - p-values ranging from 1.6×10^{-5} to 0.9 (see arXiv:2001.00821)
  - Smaller p-values were encountered for more extreme mass ratios
  - Do we expect any dependence of echoes on the mass ratio of the binary?

Data: Tentative Evidence?



- (Image from New Scientist, 20 June 2018)

Previous Work

- Most works used a gaussian initial conditions
- Only two papers account for orbital motion :
  - arXiv:1912.05419 (L. Micchi and C. Chirenti'19)
  - Phys. Rev. D, 96, 8, 084002 (Z. Mark et all'17)
- They only account for scalar waves and geodesic motion
- Goal: go beyond the geodesic approximation
- Possible differences between different mass ratios such as :
  - Which case leads to a large echo amplitude?
  - Resonances due to the orbital motion?

Teukolsky equation

- During this work we looked for solutions of the radial Teukolsky equation:

$$\Delta^{-s}\frac{d}{dr}\left(\Delta^{s+1}\frac{d\,{}_sR_{lm\omega}(r)}{dr}\right)+ \tag{1}$$
$$+\left(\frac{K^2-2is(r-M)K}{\Delta}+4is\omega r-{}_s\lambda_{lmc}\right){}_sR_{lm\omega}(r)=S_{lm\omega}(r),$$

- where $\Delta = r^2 - 2Mr + a^2$, $c \equiv a\omega$, $K \equiv (r^2 + a^2)\omega - am$
- ${}_s\lambda_{lmc}$ is a constant coming from the separation of variables
- $S_{lm\omega}(r)$ stands for a source term

Formulation: Homogeneous Solutions

- The homogeneous solutions (for s=0) are defined by their asymptotic behaviour as:

$$R^{in}_{lm\omega} \sim \begin{cases} B^{trans}_{lm\omega} e^{-ikr_*}, & \text{for} \quad r \to r_+ \\ B^{ref}_{lm\omega} \dfrac{e^{i\omega r_*}}{r} + B^{inc}_{lm\omega} \dfrac{e^{-i\omega r_*}}{r}, & \text{for} \quad r \to \infty \end{cases} \tag{2}$$

$$R^{up}_{lm\omega} \sim \begin{cases} C^{ref}_{lm\omega} e^{-ikr_*} + C^{in}_{lm\omega} e^{ikr_*}, & \text{for} \quad r \to r_+ \\ C^{trans}_{lm\omega} \dfrac{e^{i\omega r_*}}{r}, & \text{for} \quad r \to \infty \end{cases} \tag{3}$$

Formulation: Green's Function

- Follow the formulation from Mark et al. (2017)
- For the black hole case the GF reads :

$$G^{BH}_{lm\omega}(r|r') = \frac{R^{up}_{lm\omega}(r)R^{in}_{lm\omega}(r')}{W_{lm\omega}}\Theta(r-r') + \frac{R^{up}_{lm\omega}(r')R^{in}_{lm\omega}(r)}{W_{lm\omega}}\Theta(r'-r)$$

- Where $R^{in}_{lm\omega}$ and $R^{up}_{lm\omega}$ are homogeneous solutions of the Teukolsky equation
- In order to account for the reflectivity we replace $R^{in}_{lm\omega}$ by $R^{ECO}_{lm\omega}$ in the GF

$$R^{ECO}_{lm\omega}(r) = R^{in}_{lm\omega}(r) + K_{lm\omega}R^{up}_{lm\omega}(r), \tag{4}$$

Formulation: Transfer Function

- The function $K_{lm\omega}$ is computed in order to impose that the out-going amplitude of the wave at r_0 is proportional to the in-going amplitude
- It can be shown to be

$$K_{lm\omega} = \frac{B^{trans}_{lm\omega}}{C^{trans}_{lm\omega}} \bar{K}_{lm\omega} \tag{5}$$

$$\bar{K}_{lm\omega} \equiv \frac{C^{trans}_{lm\omega}}{C^{inc}_{lm\omega}} \frac{Re^{-2ikr^0_*}}{1 - (C^{ref}_{lm\omega}/C^{inc}_{lm\omega})Re^{-2ikr^0_*}}. \tag{6}$$

- R is the proportionality constant that accounts for the reflectivity

Formulation: ECO response

- Assume a scalar point charge density
- Integrate the source term against the GF
- Find that there will be two terms, the usual black hole response:

$$Z^{\infty}_{BH\,lm\omega} \propto \frac{1}{2i\omega B^{inc}_{lm\omega}} \tag{7}$$
$$\times \int_{-\infty}^{\infty} d\tau R^{in}_{lm\omega}(r_p(\tau))_0 S^{*}_{lmc}(\theta_p(\tau)) e^{-im\phi_p(\tau)} e^{i\omega t_p(\tau)},$$

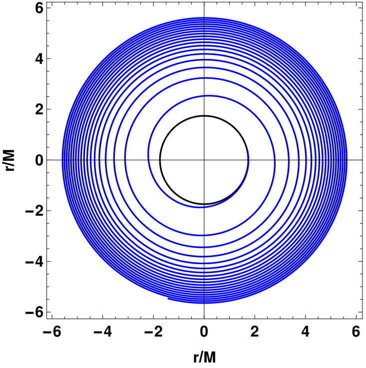
- and the echoes response, given by:

$$\tilde{\Phi}^{echo}_{lm\omega}(r) = \bar{K}_{lm\omega} \frac{e^{i\omega r_*}}{r} Z^{H}_{BH} \tag{8}$$

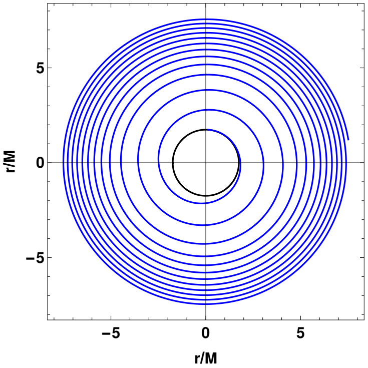
$$Z^{H}_{BH\,lm\omega} \propto \frac{B^{trans}_{lm\omega}}{2i\omega B^{inc}_{lm\omega} C^{trans}_{lm\omega}} \tag{9}$$
$$\times \int_{-\infty}^{\infty} d\tau R^{up}_{lm\omega}(r_p(\tau))_0 S^{*}_{lmc}(\theta_p(\tau)) e^{-im\phi_p(\tau)} e^{i\omega t_p(\tau)},$$

Chosen Orbits: ISCO plunge

- In order to account for the orbital effects we chose two different plunges, varying the mass of the secondary body



(b) $\mathrm{M}_1/M_2 = 1/100$



(c) $\mathrm{M}_1/M_2 = 1/20$

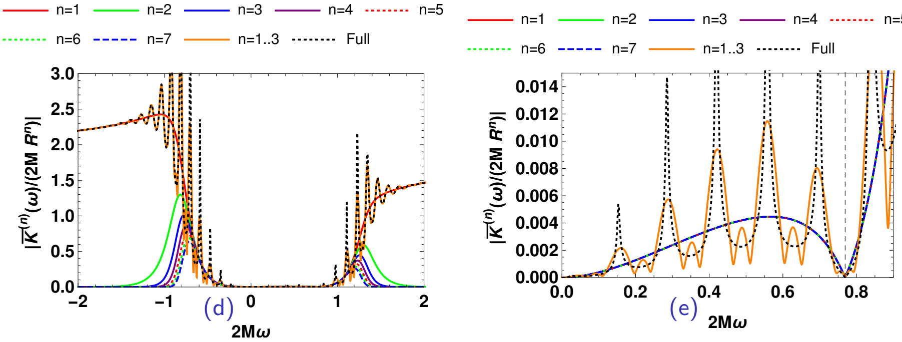
Results: $\bar{K}_{lm\omega}$ and expansions

- Looking at $\bar{K}_{lm\omega}$ plots we can see that we may be loosing some sharp peaks near in the small frequency region
- We found it to be more reliable to use an expanded version of $\bar{K}_{lm\omega}$
- The expansion is given by:

$$\overline{K}_{lm\omega} = \sum_{n=0}^{\infty} \frac{T_{BH}}{R_{BH}^{-(n-1)}} R^n e^{-2nikr_*^0}, \quad \text{if} \qquad |RR_{BH}| < 1 \tag{10}$$

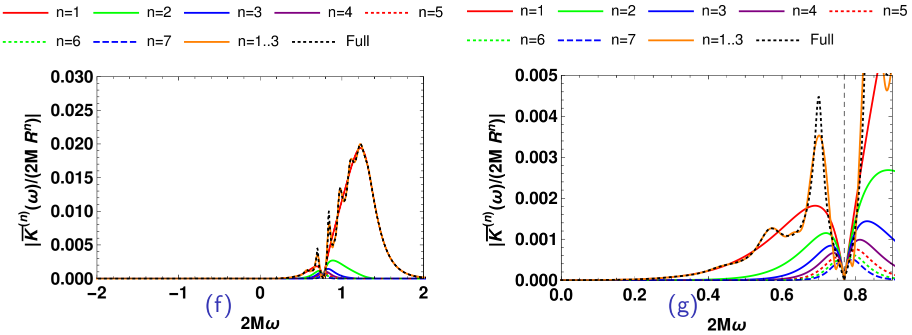
Results: $\bar{K}_{lm\omega}$ expansion

- The peaks on the full form of $\bar{K}_{lm\omega}$ come from constructive interference in the sum of the expansion terms
- Constant reflectivity : $R = 0.85$ at $r_*^0 = -50M$



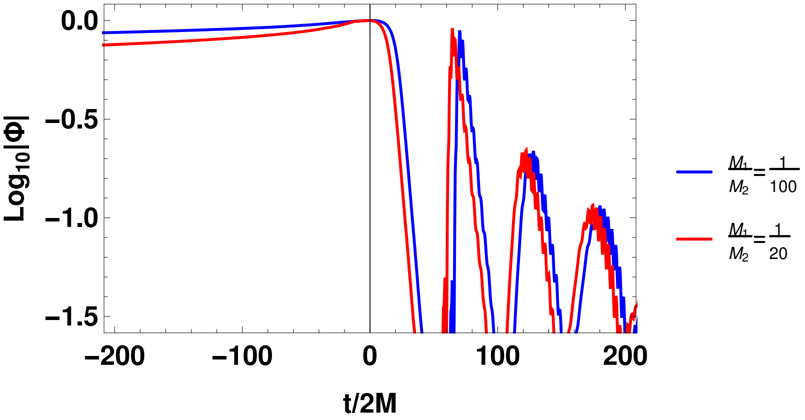
Results: $\bar{K}_{lm\omega}$ expansion

- The peaks on the full form of $\bar{K}_{lm\omega}$ come from constructive interference in the sum of the expansion terms
- Boltzmann reflectivity : $R = exp(-|k|/T)$ at $r_*^0 = -50M$, arXiv:1905.00464 by Oshita, Wang and Afshordi



Results: Echo Waveforms

- Having the waveforms at the horizon and the transfer function we can construct the echo waveforms
- For different reflectivities at $r_*^0 = -50M$ and extracted at $r = 150M$



(h) $R = 0.85$

Results: Echo Waveforms

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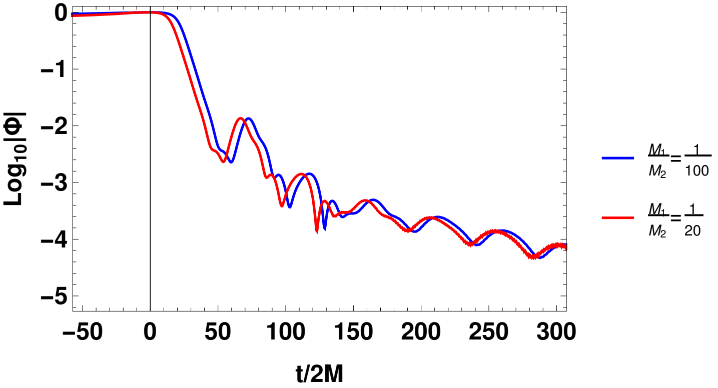
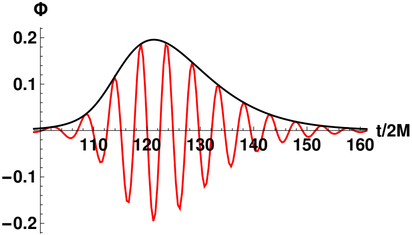


(i) $R = exp(-|k|/T)$

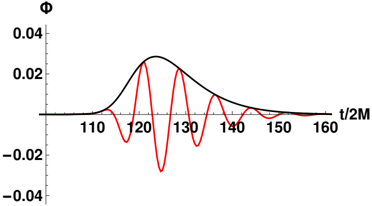
Results: Beating echoes

- Perhaps the most striking feature that we found about echoes is their beating behaviour
- This behaviour does not appear in the non-rotating case
- It originates from the asymmetry of $\bar{K}_{lm\omega}$ in the rotating case
- It becomes more evident in later echoes
- Trying to understand this beating we turned our attention for the second echo
- We saw that echoes are actually formed by two gaussian packets

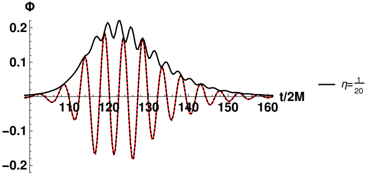
Results : Beating echoes



(j) $R = 0.85, M_1/M_2 = 1/20$: second echo from positive frequencies



(k) $R = 0.85, M_1/M_2 = 1/20$: second echo from negative frequencies



(l) $R = 0.85,\ M_1/M_2 = 1/20$: All frequencies

Conclusion and Work in Progress

- For constant R, due to the asymmetry in $\bar{K}_{lm\omega}$, negative frequencies are excited with larger amplitudes
- We found a beating behaviour which can be modelled by two superimposed gaussian wave packets
- Smaller mass ratio lead to a delay in the appearance of echoes
- Resonances of the orbital phase are negligible, agreement with analytical work in: Phys. Rev. D,100,8,084046 by Cardoso,Río and Kimura
- The next step will be to study gravitational waves, $s = -2$ (in progress)

Thank You!

UFABC

PI

McGill

Arthur B. McDonald
Canadian Astroparticle Physics Research Institute

CAPES

FAPESP